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# NUMERICAL INVESTIGATION OF A MESOSCOPIC VEHICULAR TRAFFIC FLOW MODEL BASED ON A STOCHASTIC ACCELERATION PROCESS

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## 1 ABSTRACT

In this paper a spatial homogeneous vehicular traffic flow model based on a stochastic master equation of Boltzmann type in the acceleration variable is solved numerically for a special driver interaction model. The solution is done by a modified direct simulation Monte Carlo method (DSMC) well known in non equilibrium gas kinetic. The velocity and acceleration distribution functions in stochastic equilibrium, mean velocity, traffic density, ACN, velocity scattering and correlations between some of these variables and their car density dependences are discussed.

## 2 INTRODUCTION

Mesoscopic vehicular traffic flow models are based on stochastic methods, mainly using master equations describing the time propagation of a traffic state probability function either of single cars or car clusters [1, 2, 3, 4, 5]. Especially for single car states Boltzmann like master equations are discussed in literature. Therefore those models are often called kinetic models. Mesoscopic models present the intermediate step between microscopic single car

behavior and macroscopic traffic flow. On the one hand, like in gas kinetic the huge number of parameters in a microscopic model can be reduced into mesoscopic interaction probabilities, on the other hand, dynamic macroscopic flow equations can be constructed from master equations using moment methods [6, 7, 4]. Nearly all kinetic models assume that the single car state consists of the velocity and space coordinate at any time. The interaction is restricted to a leading car pair, where only the following car changes its velocity using a stochastic jump process. Like in gas kinetic this procedure only seems valid if the mean duration of an interaction is much smaller than the mean time between any two interactions, which only holds up to moderate vehicular densities [8]. Single car measurements of the speed and acceleration time development show that the duration of acceleration changes are much shorter than the change of any other kinematic variable [9]. Therefore using an acceleration stochastic jump process seems to be the natural choice in mesoscopic car following. This idea is additionally supported by the fact that the natural driver control variable in an interaction is the acceleration.

Based on these assumptions a new mesoscopic model for the single car state probability density function in the space, velocity and acceleration variables was constructed rigorously under Markov conditions by the author reaching in an Enskog type master equation [10]. To give a guess of its applicability simple interaction functions were defined and first analytical results for a homogeneous traffic flow were discussed.

In this article a more realistic driver interaction model is constructed. Using this model, solving the master equation for a homogeneous traffic flow, acceleration and velocity distributions and important moments of the state probability density (i.e. mean velocity, velocity scattering, acceleration scattering) depending on the car density in stochastic equilibrium are calculated.

Section 3 summarizes the model and its equations. In section 4 the interaction model is introduced. For solving the master equation a numeric procedure borrowed from gas kinetic is used and shortly described in section 5. In Section 6 the results are shown and discussed.

### 3 SUMMARY OF THE BASIC MODEL

In this section the basic homogenous traffic flow model equation derived in [10] and its requirements are shortly summarized. At time  $t$  the leading car has velocity  $\bar{v}$  and acceleration  $\bar{a}$  and the following car has velocity  $v$  and acceleration  $a$  neglecting the spatial coordinates due to homogeneity. This pair of cars is called a *leading* car pair. The interaction between the cars in a given leading car pair is assumed to be a Markov jump process in the acceleration variable  $a$  of the following car. Therefore the time propagation of

the car pair state probability density fulfills the Feller-Kolmogorov equation. Introducing a vehicular chaos ansatz [11, 10], i.e. a leading car pair is only distance correlated, and assuming that an interaction does not depend on the acceleration of the leading car  $\bar{a}$ , because its value can not be determined by the driver of the following car, the spatial homogenous single car probability density  $f$  is given by the solution of

$$\frac{\partial f}{\partial t} + a \frac{\partial f}{\partial v} = \int_{\bar{v}, a'} d\bar{v} da' \tilde{f}(\bar{v}, t) \cdot \{\Sigma(a|v, \bar{v}, a', \mathbf{m}_f(t))f(v, a', t) - \Sigma(a'|v, \bar{v}, a, \mathbf{m}_f(t))f(v, a, t)\} . \quad (1)$$

$f(v, a, t)dv da$  is the probability to find a single vehicle at velocities between  $v$  and  $v + dv$  and accelerations between  $a$  and  $a + da$  at time  $t$  and  $\tilde{f}(v, t) = \int_a f(v, a, t) da$  is its reduced density. The weighted interaction density  $\Sigma$  includes the rate  $Q$  of a single interaction for a given car pair state  $(v, a, \bar{v})$  and the interaction strength probability density  $\sigma$  weighted by a distance correlation function  $D$ , i.e.

$$\Sigma(a|v, \bar{v}, a', \mathbf{m}_f(t)) = \int_{h_{\min}}^{\infty} \sigma(a|h, v, \bar{v}, a')Q(h, v, \bar{v}, a')D(h|v, a', \mathbf{m}_f(t)) dh . \quad (2)$$

Here  $D$  is equal to the conditioned probability density of the distance  $h$  depending on the state of the following car and the probability density  $f$  or some moments of  $f$  called moment vector  $\mathbf{m}_f$  at time  $t$ . In this paper, as described below, the conditioned distance density is chosen in such a way that the car density  $K$  is included into  $\mathbf{m}_f$  as an constant, additional element. Note that  $D$  vanishes for  $h$  lower than the minimal distance  $h_{\min}$  in a leading car pair at rest. A further specification of these functions will be done in section 4.

Eq. 1 shows the typical structure of a stochastic rate equation. The total rate of change of  $f$  at the state  $(v, a)$  at time  $t$  is equal to the difference of probability inflow in this state and outflow out of this state. The vehicular chaos assumption, analog to gas kinetic, leads in the typical Boltzmann like product form of  $f$  on the right side. Note that in this paper a car following model without any overtaking is discussed.

Additional to the equation an initial condition and boundary conditions in  $v$  must be added. The boundary conditions introduced here are described in [10] and ensure that negative velocities  $v < 0$  are impossible. In the case that a braking car reaches  $v = 0$ , independent on the state of its leading car, the driver changes the acceleration to  $a = 0$ . The interaction described in the next section is distance oriented, and therefore does not describe traffic flow at low car densities, i.e. the free flow regime. This regime is included by a maximal velocity  $w$  via a high speed boundary condition. An accelerating

car reaching  $v = w$  changes the acceleration to  $a = 0$  to hold this velocity, independent on the behavior of the leading car. Because of this boundary condition velocity and acceleration scattering do not occur in this regime in contrast to measurements where slight scattering is always present. To relax this unrealistic behavior, the boundary condition at  $v = w$  must be changed into a relative velocity dependent interaction at high velocities or large distances as new unpublished developments on the model show. Further work on this field is under way.

Because in this paper only the stochastic equilibrium is discussed, the concrete form of the initial condition is not important. Here a Gaussian velocity distribution with vanishing acceleration is used. The simplest way to reach the stochastic equilibrium density is to calculate the time propagation of a homogeneous traffic flow, bearing in mind that in reality it remains not homogeneous during this process.

## 4 DESCRIPTION OF THE CAR PAIR INTERACTION

In this section the interaction and distance correlation functions, defined in Eq. 2 are concretized. Here a threshold or action point interaction often described in literature for microscopic or mesoscopic models [12, 13] is used. For simplicity the interaction is restricted to a single velocity dependent deterministic threshold for all cars. Therefore the driving behavior must be conservative to avoid accidents.

### 4.1 The Interaction Rate

In the action point model, interactions only occur on velocity dependent distance thresholds, redefining the interaction rate  $Q$  compared to those described in [10]. In a given leading car pair the following car with velocity  $v$  interacts, i.e. changes its acceleration state, if the distance  $h$  to the leading car is equal to an action point or threshold distance  $H(v)$ . So  $h = h(t)$  and  $H(t) = H(v(t))$  are both functions of time  $t$ . Staying at time  $t$  shortly before the threshold, i.e.  $h(t) - H(t) \rightarrow 0^-$  or  $h(t) - H(t) \rightarrow 0^+$ , assuming an interaction at time  $t + \tau$ , where  $\tau$  is infinitesimal small,

$$\begin{aligned} H(t + \tau) &= H(t) + H'(v) \cdot a\tau \quad \text{with} \quad H'(v) = \frac{dH}{dv}, \\ h(t + \tau) &= h(t) + (\bar{v} - v)\tau \end{aligned} \tag{3}$$

are the threshold and distance values directly after this interaction up to the first order in  $\tau$ . An acceleration interaction  $a > 0$  is given by  $h(t) - H(t) < 0$

and  $h(t + \tau) - H(t + \tau) \geq 0$ , where a de acceleration interaction,  $a < 0$ , is given by  $h(t) - H(t) > 0$  and  $h(t + \tau) - H(t + \tau) \leq 0$ . Inserting this into Eqs. 3, lead to necessary conditions for acceleration  $\bar{v} - v - H'(v) \cdot a > 0$  and de acceleration  $\bar{v} - v - H'(v) \cdot a \leq 0$ . Because an interaction occurs in a deterministic way when the distance between the two cars passes the threshold, the probability of the acceleration interaction  $P^+(\tau)$  and the de acceleration interaction  $P^-(\tau)$  are given by

$$\begin{aligned} P_+(\tau) &= \Theta(h(t + \tau) - H(t + \tau))\Theta(\bar{v} - v - H'(v) \cdot a) , \\ P^-(\tau) &= \Theta(H(t + \tau) - h(t + \tau))\Theta(-(\bar{v} - v - H'(v) \cdot a)) , \end{aligned} \quad (4)$$

where  $\Theta(x)$  is the Heaviside unit step function. The probability to have an interaction until  $\tau$  now is  $P^+(\tau) + P^-(\tau)$ , and its derivative at  $\tau = 0$  with Eqs. 3 is the interaction rate

$$Q(h, v, \bar{v}, a, \bar{a}) = |\bar{v} - v - H'(v) \cdot a| \delta(h - H(v)) , \quad (5)$$

where  $\delta(x)$  is the Dirac distribution. A linear velocity dependence

$$H(v) = h_{\min} + \alpha v \quad (6)$$

with some given constant  $\alpha$  specified below is used, bearing in mind that this is a conservative driver behavior ansatz at high velocities [12].

## 4.2 The Interaction Strength

The strength of an interaction is described by the probability density of an acceleration or de acceleration change in analogy to the differential cross section in gas kinetic. Because only a single interaction threshold is used especially in the de acceleration process one has to ensure  $h \geq h_{\min}$  to avoid accidents. This is only possible by defining the de acceleration change  $a^-$  as the total braking value of the following vehicle with given velocity  $v$  until it stops in the distance  $H(v) - h_{\min} + \bar{h}$ , where  $\bar{h}$  is the minimum braking distance of the leading vehicle at velocity  $\bar{v}$  with maximum braking value  $\bar{a} = a_{\min} < 0$ :

$$a^- = \frac{-v^2}{2(H(v) - h_{\min} + h^*)} \quad \text{with} \quad \bar{h} = \frac{\bar{v}^2}{2|a_{\min}|} . \quad (7)$$

The acceleration change  $a^+$  depends strongly on the actual velocity  $v$  of the car. Where at very low velocities the acceleration value increases, at higher velocities it decreases and vanishes near by the maximum velocity  $w$  [14].

This is modeled using a linear increasing function from  $a^+ = a_0 > 0$  at  $v = 0$  to  $a^+ = a_{\max} > 0$  at  $v = v_m \ll w$  followed by a linear decreasing function starting at  $v = v_m$  and reaching  $a^+ = 0$  at  $v = w$ , which is a simple caricature of measured driver behavior:

$$a^+ = \begin{cases} \frac{w-v}{T}, & v \geq v_m, \\ a_0 + \frac{a_{\max}-a_0}{v_m}v & \text{with } a_{\max} = \frac{w-v_m}{T}, \quad v < v_m. \end{cases} \quad (8)$$

Individual driver behavior shows some scattering in the acceleration strength. Here for simplicity and to minimize the number of parameters this scattering is avoided. Therefore the interaction strength density at  $h = H(v)$  is given by

$$\sigma(a|H(v), v, \bar{v}, a', \bar{a}) = \begin{cases} \delta(a - a^+), & \bar{v} - v - H'(v) \cdot a' > 0, \\ \delta(a - a^-), & \bar{v} - v - H'(v) \cdot a' \leq 0, \end{cases} \quad (9)$$

where the decision of accelerating or de accelerating depends on the sign of  $\bar{v} - v - H'(v) \cdot a'$  as is discussed above.

It is well known that driver behavior in acceleration and de acceleration depends on the relative velocity between the leading and the following car [15, 16]. This behavior homogenizes the traffic flow. Comparing the simulation results (see below) to those found in literature show a qualitative agreement for velocity dependent quantities. In contrast to this the acceleration distribution shows significant deviations mainly reducible to the lack of relative velocity dependence of the acceleration change in the model [17]. To integrate such an effect into the interaction, a second, more distant threshold must be used additionally to the profile used here. On this threshold a relative velocity dependent acceleration change can be defined. Further work to the influence on the resulting quantities and comparison to measured data is underway by the author.

### 4.3 The Spatial Correlation

The spatial correlation is given by the following driver dependent distance probability density  $D(h|a, v, \mathbf{m}_f)$ . For the single threshold interaction model the sign of the acceleration  $a$  of the following car is equal to the sign of  $h - H(v)$ . Restricting to the case where the velocity dependence is only given via  $H(v)$  the following ansatz can be used for acceleration and de acceleration

$$D(h|a < 0, v, \mathbf{m}_f) = D(h|h \leq H(v), \mathbf{m}_f) = \frac{\tilde{D}(h|\mathbf{m}_f)}{\int_{h_{\min}}^{H(v)} \tilde{D}(h|\mathbf{m}_f) dh},$$

$$D(h|a \geq 0, v, \mathbf{m}_f) = D(h|h > H(v), \mathbf{m}_f) = \frac{\tilde{D}(h|\mathbf{m}_f)}{\int_{H(v)}^{\infty} \tilde{D}(h|\mathbf{m}_f) dh}. \quad (10)$$

The second equality is given by the conditioned probability standard theorem [18]. Here  $\tilde{D}(h|\mathbf{m}_f)$  is the driver unconditioned distance probability density with parameters depending on moments of the unknown  $f$ . By inserting Eq. 10 into Eq. 13 of paper [10] it is shown that this equation is fulfilled identically. Therefore the car density  $K$  is a free parameter of the problem, which in the case of equilibrium can be chosen as time-constant. It is included into  $\mathbf{m}_f$ . Normally distance measurements are done locally resulting in time headway distributions, which are often approximated by gamma densities [19]. Therefore in this article a gamma density is also used neglecting possible differences between time headway density and distance density

$$\tilde{D}(h|r, \lambda) = \frac{\lambda}{\Gamma(r)} (\lambda(h - h_{\min}))^{r-1} e^{-\lambda(h-h_{\min})} \Theta(h - h_{\min}), \quad (11)$$

where  $\Gamma(x)$  is the standard gamma function [20]. Depending on the parameter values  $\lambda(\mathbf{m}_f) > 0$  and  $r(\mathbf{m}_f) > 1$  a wide spread of shapes can be constructed including Gaussian, exponential and in the limit case delta densities. These parameters are specified by the following conditions. The mean distance, i.e. the mean value of  $\tilde{D}$ , is equal to the inverse of the car density  $K$  and the scattering of  $\tilde{D}$  here is modeled to be proportional to the mean velocity of all cars  $V$  (with constant  $\beta$ ). The second ansatz seems to be feasible, because with increasing mean velocity and therefore decreasing car density the distance scattering increases also. These two conditions together with Eq. 11 result in  $r = (K^{-1} - h_{\min})^2 / (\beta V)^2$  and  $\lambda = (K^{-1} - h_{\min}) / (\beta V)^2$ . So the moment vector dependence  $\mathbf{m}_f$  is reduced to the mean velocity  $V$  and the car density  $K$ . Inserting Eq. 11 into 10, bearing in mind that due to Eq. 5 and the interaction Eq. 2 only the value at  $h = H(v)$  is needed, result in

$$\begin{aligned} D(H(v)|a \geq 0, v, K, V) &= \frac{(\lambda(H(v) - h_{\min}))^r e^{-\lambda(H(v)-h_{\min})}}{(H(v) - h_{\min})(\Gamma(r) - \gamma(r, \lambda(H(v) - h_{\min})))}, \\ D(H(v)|a < 0, v, K, V) &= \frac{(\lambda(H(v) - h_{\min}))^r e^{-\lambda(H(v)-h_{\min})}}{(H(v) - h_{\min})\gamma(r, \lambda(H(v) - h_{\min}))}, \end{aligned} \quad (12)$$

where  $\gamma(r, x)$  is the incomplete gamma function [20]. So the weighted interaction density, Eq. 2, is totally specified by Eqs. 5, 9 and 12.

## 5 NUMERICAL REMARKS

The solution of the master equation together with the interaction functions specified is only possible numerically. Like in gas kinetic this type of integro-differential equation is hard to implement on a computer system using standard integration and differentiation approximation rules. Nevertheless, there are stochastic solution methods called *direct simulation Monte Carlo* (DSMC) well established in non equilibrium gas kinetic for the Boltzmann equation, which after some modifications can be used following the ideas of Nanbu, 1981 [21]. Only a short description will be given, a detailed analysis is published elsewhere [22, 23].

In DSMC methods the probability density  $f$  is approximated by a discrete measure, here a number  $N$  of stochastic cars. Each car has a velocity  $v$  and an acceleration  $a$ . The simulation procedure is split into two parts over a discrete time interval  $\Delta t$  small against all characteristic timescales of the process. In the first part, all cars are accelerated or de accelerated without any interaction. In the second part, for each car there is done a decision, whether it interacts with an other one due to the total interaction probability  $\Delta t \cdot \int_a \Sigma(a|v, \bar{v}, a', \mathbf{m}_f) da$ . Any interaction is constructed by changing the acceleration value of the following car using Eq. 2. Then again all cars change their velocities due to their actual acceleration values. The simulation results in this paper are based on  $N = 100$  cars, which are followed in time until the stochastic equilibrium density  $f_{\text{eq}}(v, a)$  is reached. The process is repeated 100 times to reduce numerical scattering. In each run the simulation stops, when changes in the distribution are negligible and the mean acceleration nearly vanishes.

## 6 DISCUSSION OF THE RESULTS

In this section the velocity and acceleration distribution together with some of their moments like mean values, scattering, skewness and correlations depending on the car density  $K$  in stochastic equilibrium are discussed. To reduce the number of parameters some scaling is introduced. There are two interaction independent parameters  $h_{\text{min}}$  and  $w$  in the model, which together with the acceleration scale  $w^2/h_{\text{min}}$  are used in the discussion. In this section scaled variables and functions are marked by a prime. The numerical solution is done using the following parameter values:  $\alpha' = 10$ ,  $a'_{\text{min}} = -0.013$ ,  $v'_m = 0.125$ ,  $T' = 86$  and  $\beta' = 0.57$ , which can be applied for typical values of  $h_{\text{min}} \approx 7\text{m}$  and  $w \approx 40\text{m/s}$ . So for example the value of  $\alpha$  is defined in such a way that the threshold is approximately equal to half of the velocity in units of km/h.

Figs. 1 and 2 show the typical fundamental diagram for the mean velocity

$$V = E[v] = \int_{v,a} v f_{\text{eq}}(v, a) dv da \quad (13)$$

and traffic density  $q = V \cdot K$  depending on the car density  $K$ . The early decrease of the mean velocity is due to the linear threshold behavior, which at high velocities is a conservative driving approximation.

Defining the scattering of a variable  $x$  as  $\sigma_x^2 = E[(x - E[x])^2]$  using definition Eq. 13 for  $E[x]$  analogously, figs. 3 and 4 show the car density dependence of the velocity and acceleration scattering resp.. Especially the acceleration scattering, often called acceleration noise (ACN), shows the same behavior as measurements predict [24]. There at very low car densities, in the free flow region, only a small amount of ACN occurs, which vanishes for  $K \rightarrow 0$  due to the boundary condition at  $v = w$ . As mentioned above the free flow part is in contrast to experimental results, where typical values of  $0.3\text{m/s}^2$  occur. With increasing car density, in the area of partly constrained traffic, there is a huge increase of ACN. At high densities – the so called constrained traffic area – ACN decreases strongly, because all driver are obliged to have nearly the same behavior. The velocity scattering shows the same shape. Therefore a positive correlation between both moments can be expected. This is shown in fig. 5 using the definition of centralized moments  $\text{Corr}(x^n, y^m) = E[(x - E[x])^n (y - E[y])^m] / (\sigma_x^n \sigma_y^m)$  for any two random variables  $x$  and  $y$ . With increasing car density the correlation  $\text{Corr}(v^2, a^2)$  reaches a maximum, similar to that of the velocity and acceleration scattering. At higher car densities the decrease of scattering is shifted between both quantities resulting in a slight decrease in the correlation. In contrast to the scattering correlation in fig. 5, the correlation between mean velocity and ACN, fig. 6, is negligible as is obtained for simple interaction models introduced in [10].

The skewness  $S = E[(x - E[x])^3] / \sigma_x^3$  is a measure for the symmetry of a distribution. Here fig. 7 shows the skewness of the velocity distribution. At small car densities there is a strong increase from large negative values to zero, which re give the transition from a delta peaked velocity density at  $v = w$ , over a left skewed distribution in the transition area, until a symmetric distribution at high car densities, where the boundary condition at  $v = w$  is not significant any longer.

In fig. 8 the density dependence of the equilibrium mean interaction rate defined by

$$\nu = \int_{\bar{v}, v, a, a'} \Sigma(a|v, \bar{v}, a', \mathbf{m}_f) f_{\text{eq}}(v, a') \tilde{f}_{\text{eq}}(\bar{v}) d\bar{v} dv da da' \quad (14)$$

is plotted. As expected the interaction rate increases with increasing car density starting at  $K' \approx 0.1$ , where the partly constraint traffic flow begins. Note that there is no decrease at high car densities, because even when all cars are stopped in a traffic jam the drivers want to accelerate, which produce a lot of small valued interactions.

Fig. 9 shows velocity distributions for different  $K$  values, which are more or less Gaussian like, as is well known from measurements [4]. The analog acceleration distributions are shown in fig. 10. They are strongly bimodal reflecting the interaction strength density Eq. 9, used in this simulation. Note that measured distributions are much broader in zero direction [17]. This difference is possibly due to the single threshold interaction model used here as mentioned above.

## 7 CONCLUSIONS

- (1) A spatial homogeneous vehicular traffic flow model based on a stochastic master equation of Boltzmann type in the acceleration variable is applied to a concrete interaction model. For solution a modified DSMC method well known in non equilibrium gas kinetic is used. The velocity and acceleration distribution functions in stochastic equilibrium and some of their moments and correlations are discussed.
- (2) The fundamental diagrams, i.e. the car density dependence of the mean velocity and the traffic density, are in qualitative agreement with other models or measurements. Calculated ACN shows the same qualitative behavior as experiments predict. There is a huge increase of ACN at the transition from free to partly constraint traffic. The velocity scattering is positive correlated to the ACN.
- (3) There is no significant skewness in the velocity distributions at higher car densities. The velocity distributions show a symmetric and more or less Gaussian distributed behavior. There is only a very small correlation between the mean velocity and the ACN in a traffic flow. Acceleration distributions are bimodal and strongly peaked in contrast to experimental work. Here additional investigations especially the introduction of a relative velocity dependent acceleration strength in an interaction have to take place.

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## FIGURE CAPTIONS

Fig. 1: Scaled mean velocity as a function of the scaled car density.

Fig. 2: Scaled traffic density as a function of the scaled car density.

Fig. 3: Scaled velocity scattering as a function of the scaled car density.

Fig. 4: Scaled ACN as a function of the scaled car density.

Fig. 5: Scattering correlation between  $\sigma_v$  and ACN as a function of the scaled car density.

Fig. 6: Correlation between mean velocity  $V$  and ACN as a function of the scaled car density.

Fig. 7: Skewness of the velocity distribution as a function of the scaled car density.

Fig. 8: Scaled mean interaction rate as a function of the scaled car density.

Fig. 9: Velocity distributions for different values of the scaled car density.

Fig. 10: Acceleration distributions for different values of the scaled car density.

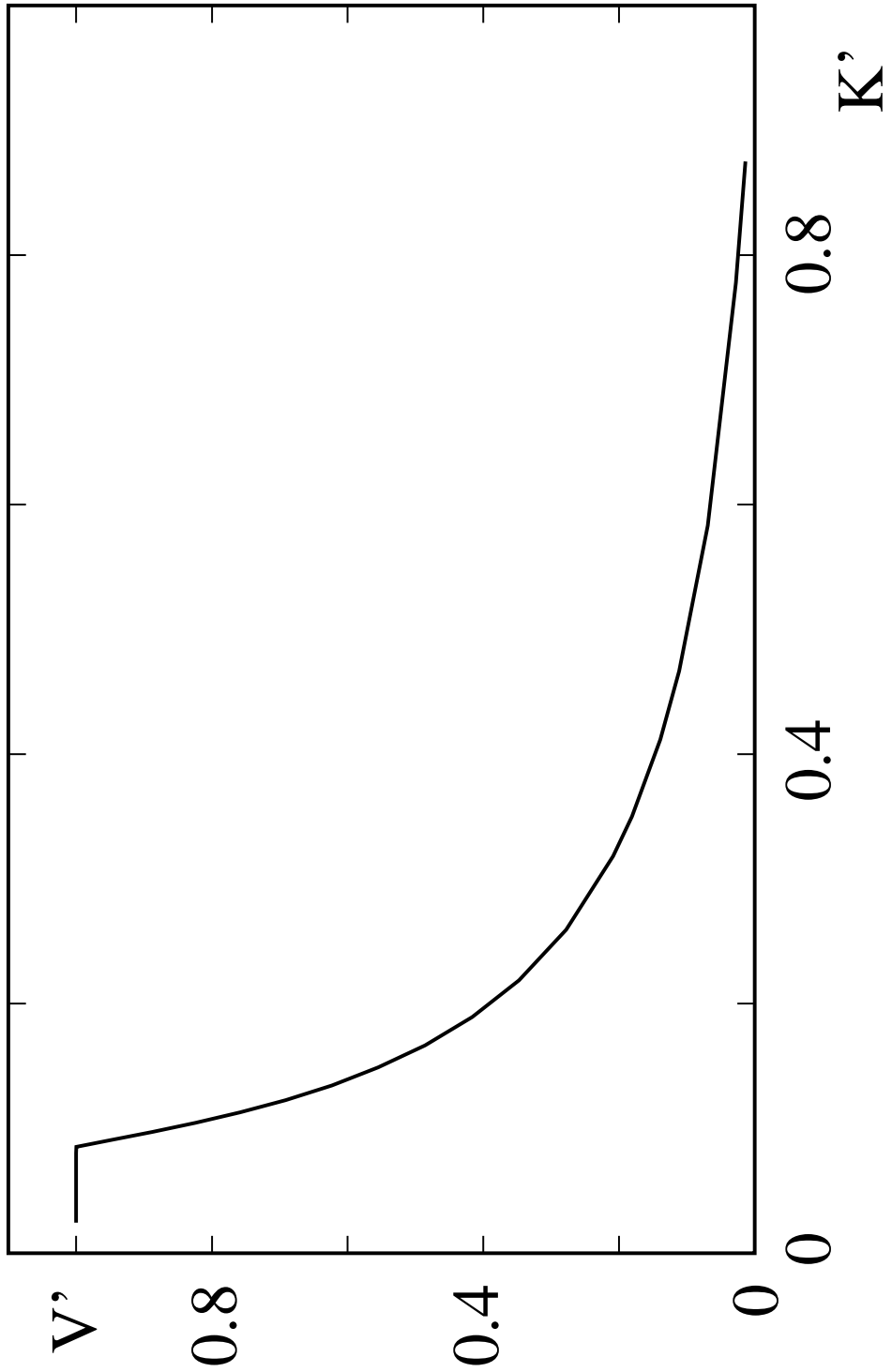


Figure 1:

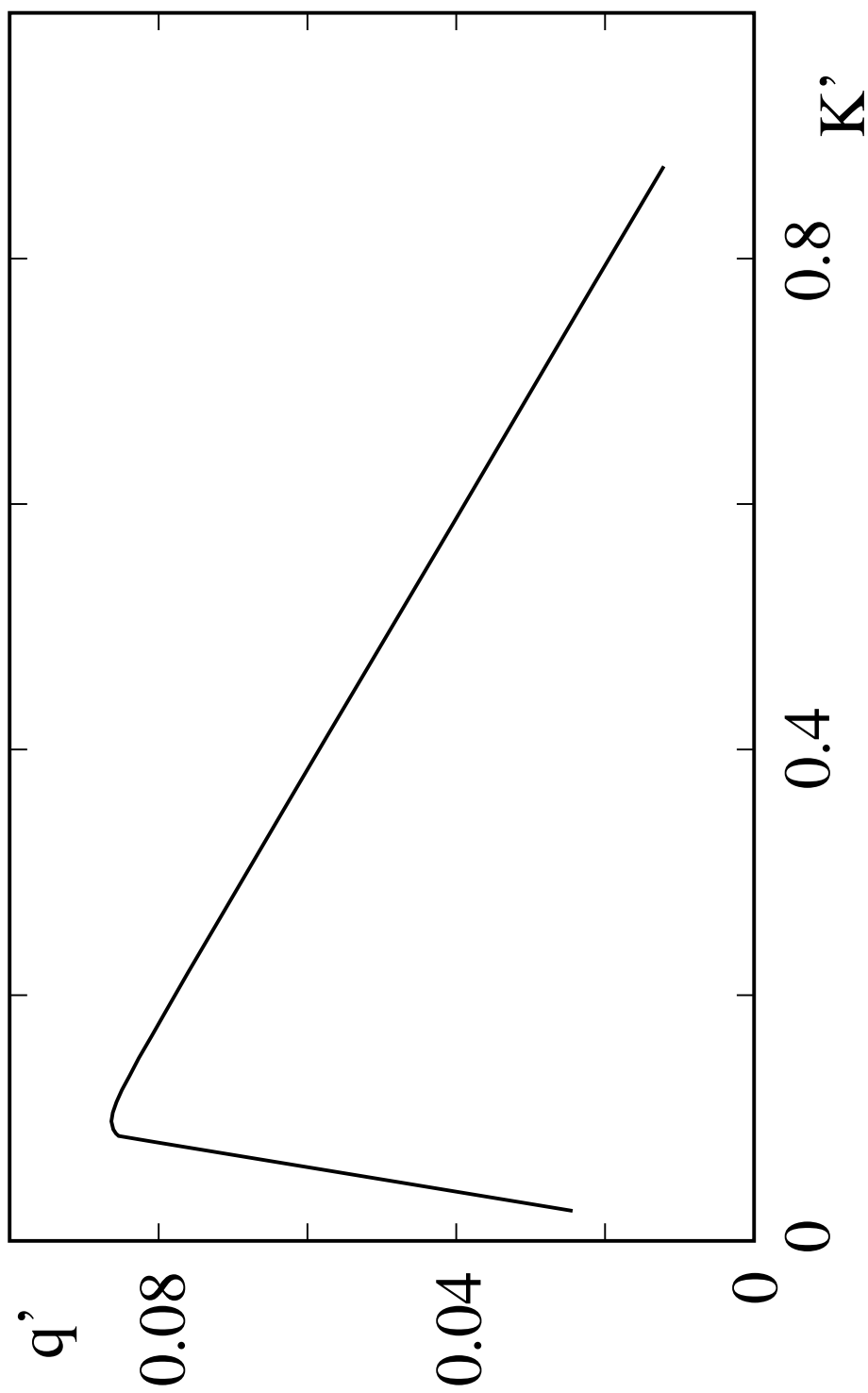


Figure 2:

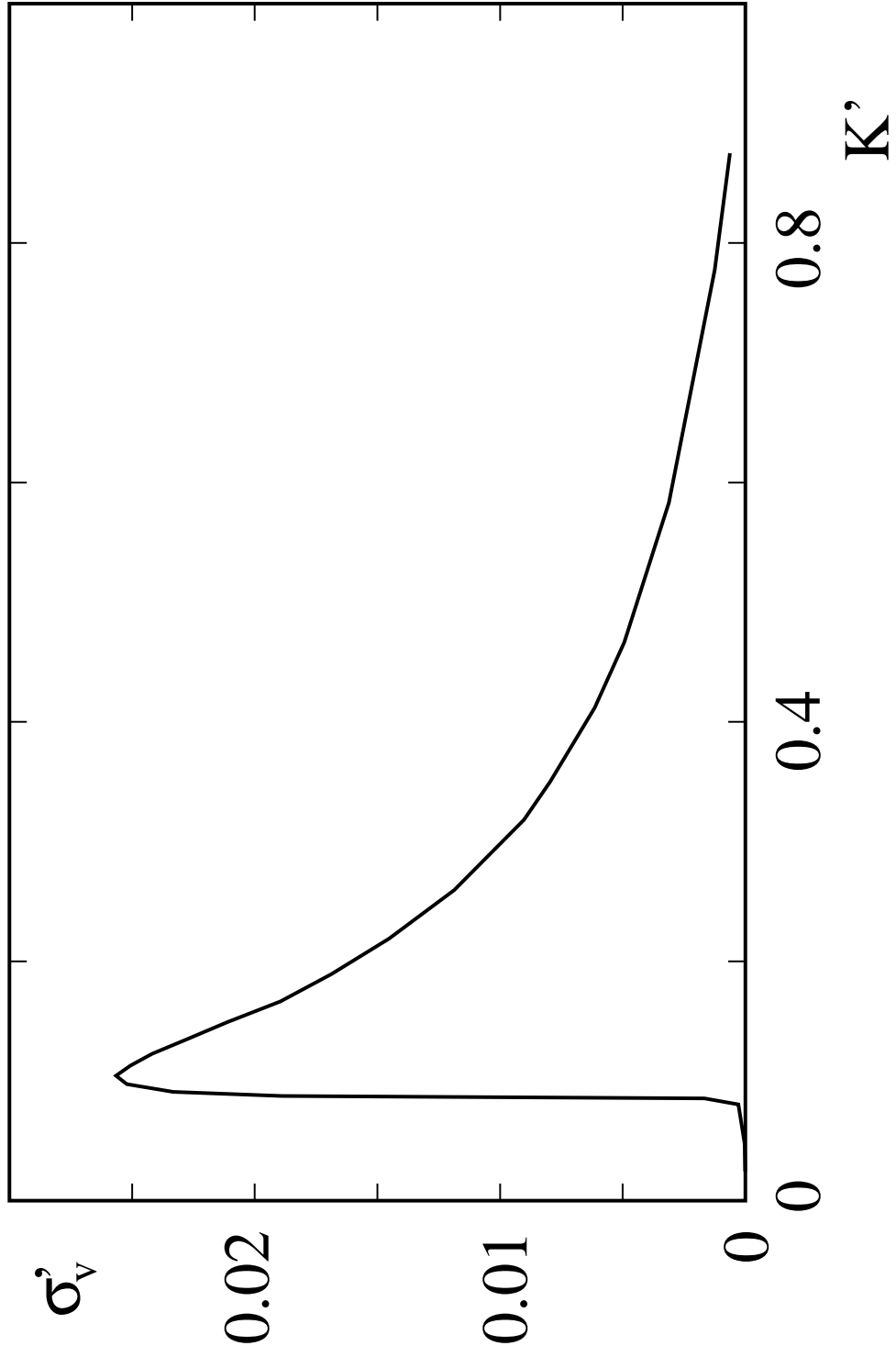


Figure 3:

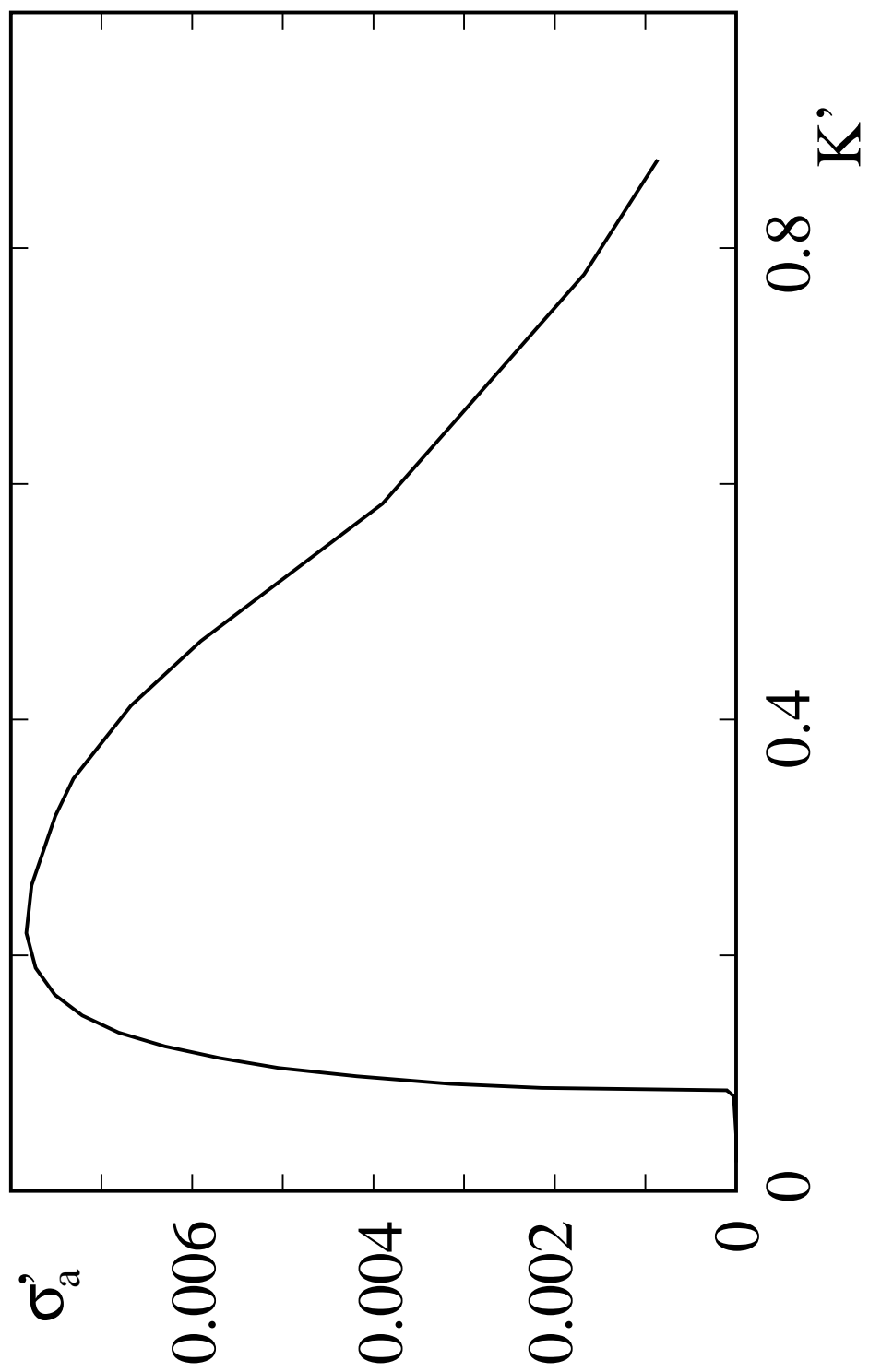


Figure 4:

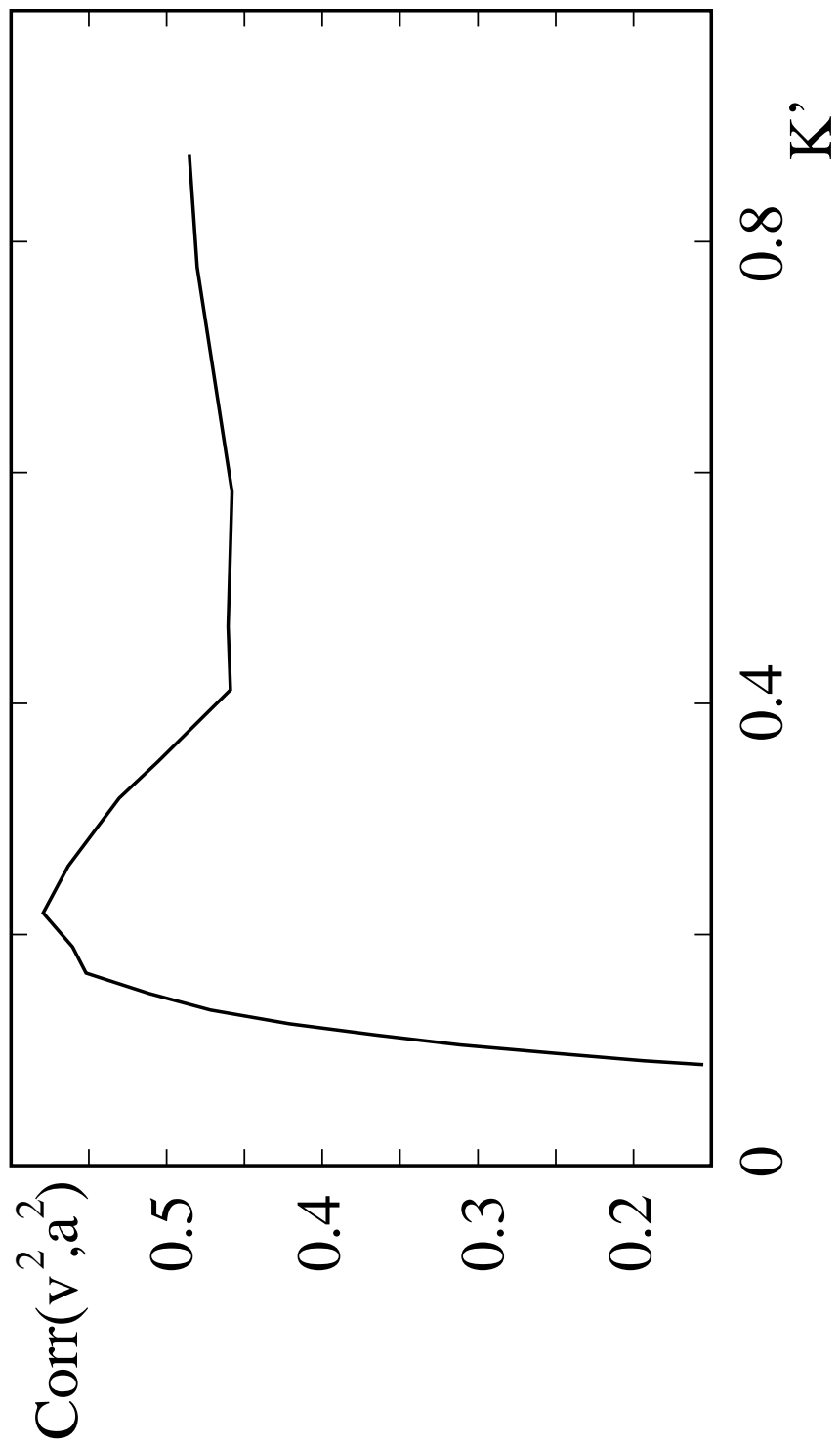


Figure 5:

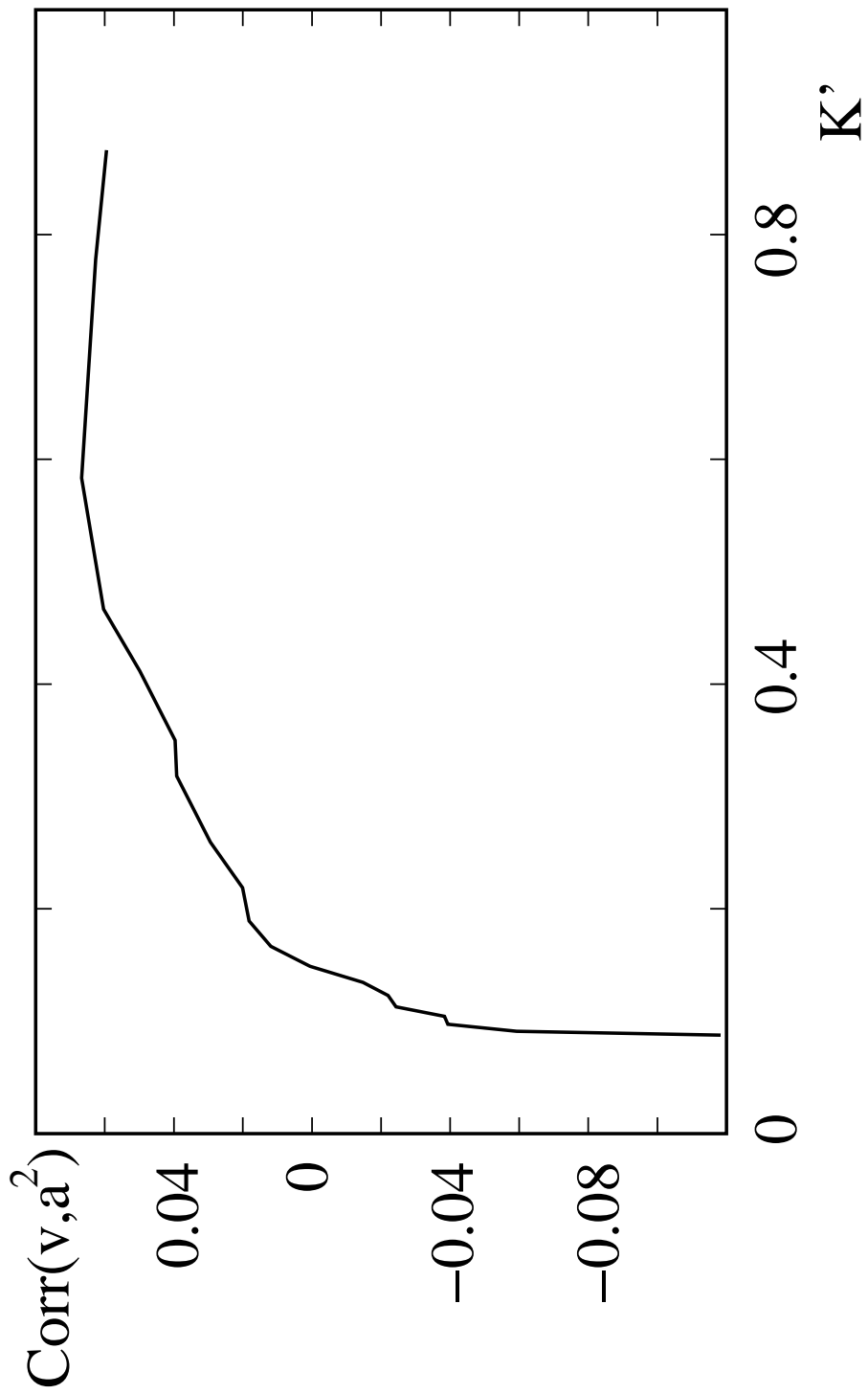


Figure 6:

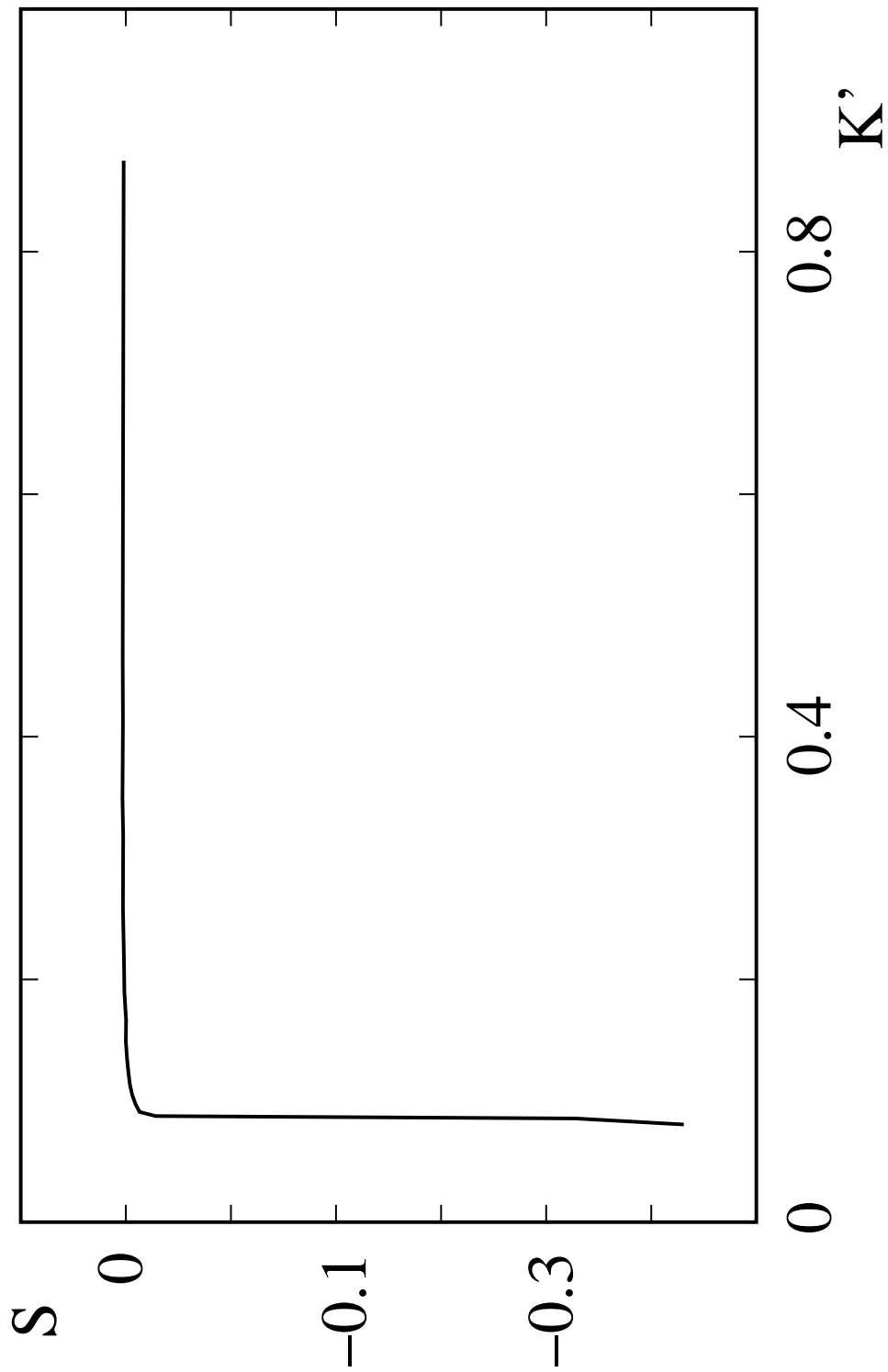


Figure 7:

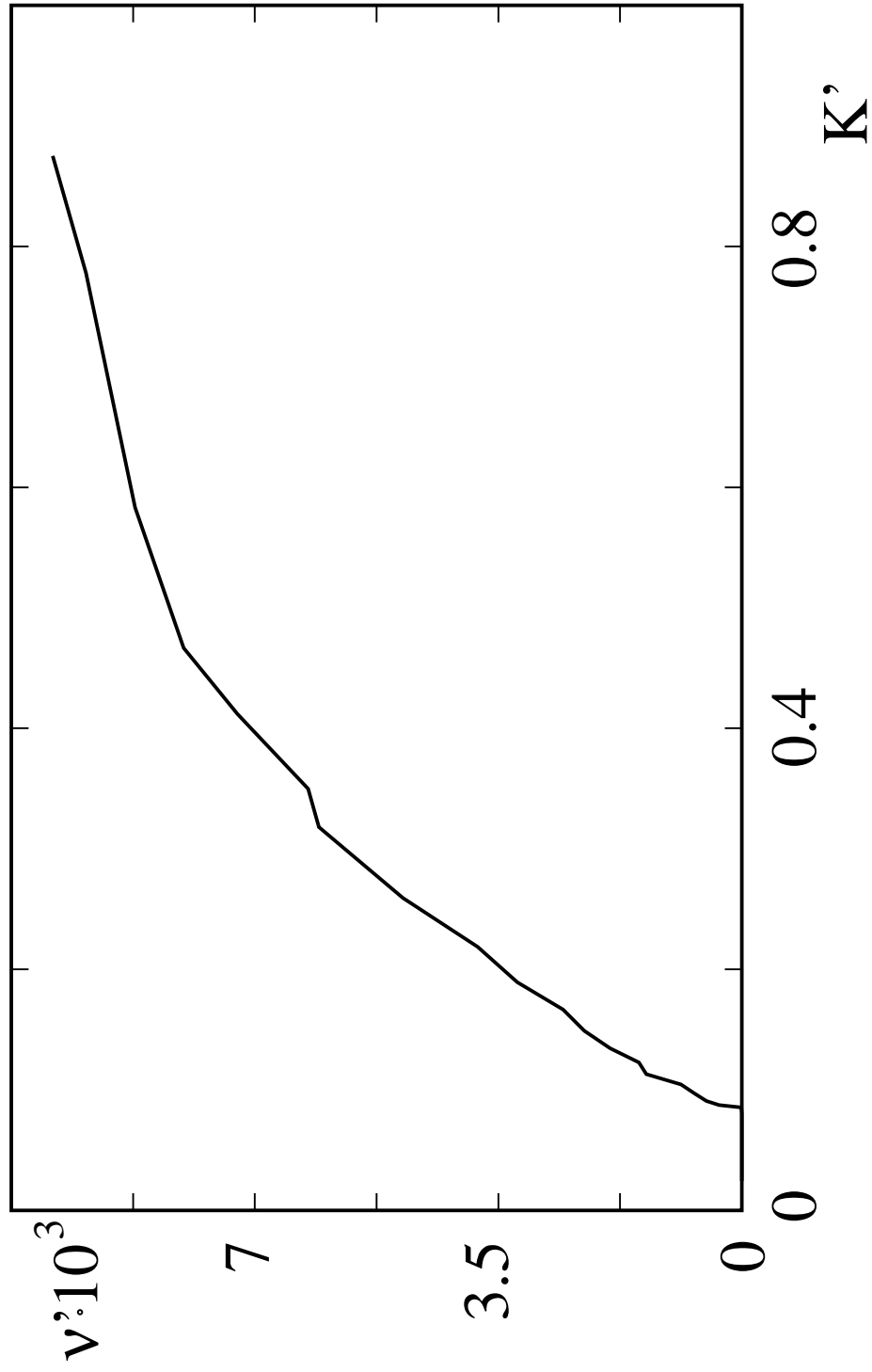


Figure 8:

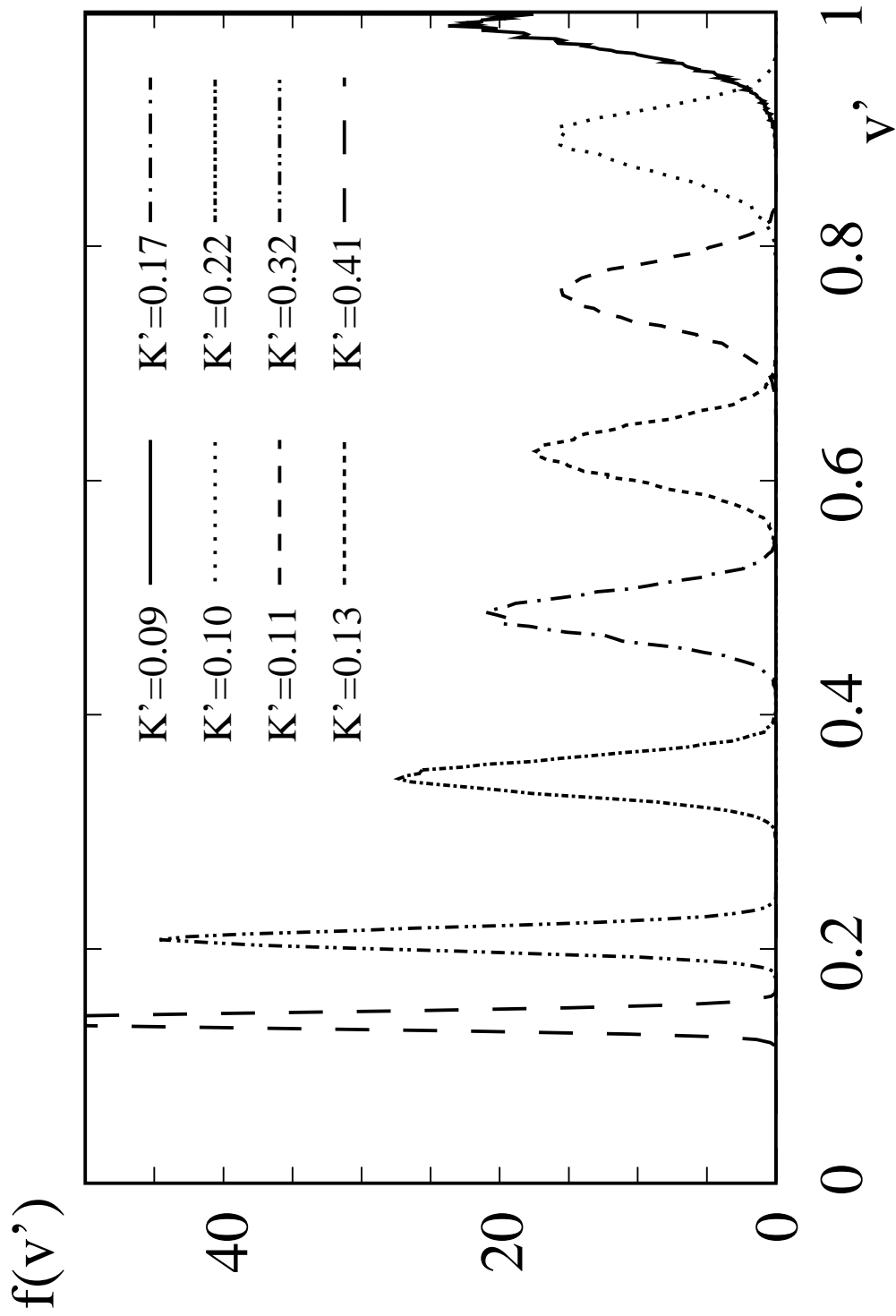


Figure 9:

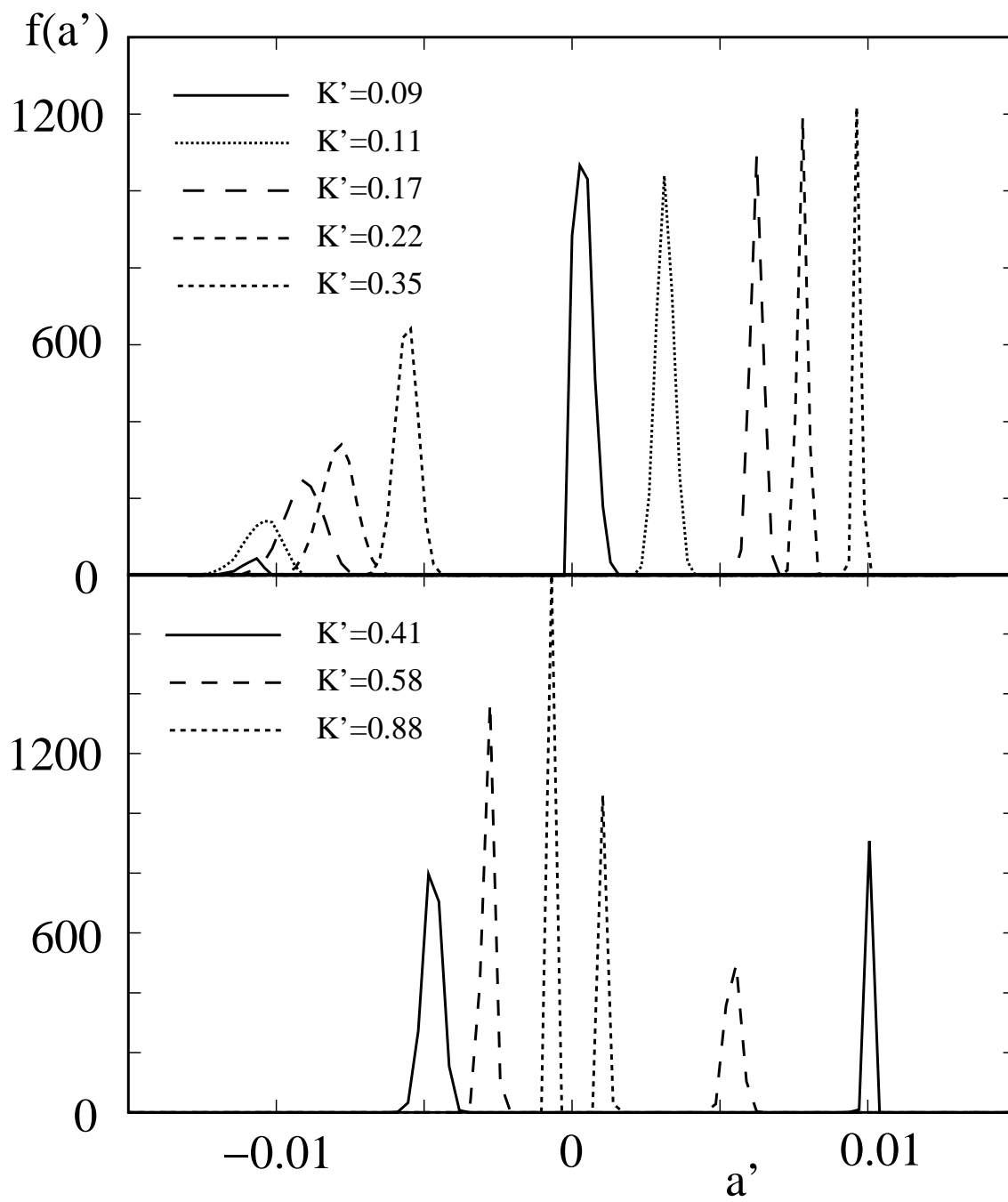


Figure 10: