

The Direct Simulation Monte Carlo Method applied to a Boltzmann-like Vehicular Traffic Flow Model

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Abstract

In this paper a direct simulation Monte Carlo (DSMC) method is applied to a spatial homogeneous mesoscopic vehicular traffic flow model, based on a Boltzmann-like master equation. In contrast to gas kinetics, where in a collision a velocity jump change occurs, the interaction now changes the acceleration value of the following car in a leading car pair. There are no conservation laws in a single interaction. Therefore the Bird simulation scheme seems not to be the right choice for the approximation of the interaction integral. It is shown, that a Nanbu like scheme is natural for this process. To avoid the typical double loop computational effort of the Nanbu scheme, a sampling algorithm developed by Babovsky is applied. Several car interaction profiles are examined and their resulting stochastic equilibrium solutions are discussed. First, simple interaction profiles are used to compare the simulation results with analytic calculated velocity distributions showing excellent agreement. Second, a realistic distance threshold interaction profile is applied to the simulation and the results are shown to be in qualitative agreement with measured traffic flow data. The simulation procedure seems to be applicable to study the influence of different interaction profiles to the macroscopic vehicular traffic flow quantities in stochastic equilibrium.

Key words: Vehicular traffic flow, kinetic modeling, direct simulation Monte Carlo scheme, Boltzmann equation

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1 Introduction

The success of the quantitative physical method for describing effects or measurements leads to its growing popularity in other scientific and engineering fields, so in vehicular traffic flow theory also. Especially the description of interacting many body systems in the framework of physical concepts finds increasing prevalence. As a consequence physical computer solving methods are adapted to those established models. Vehicular traffic flow theory was found with the first experimental data from road traffic, which resulted in flow variable dependences, independent of the concrete probe measured [1]. Therefore an objective understanding seems to be possible and leads to several model approaches often based on computer simulation methods. Up to now, there is no model describing all typical measured quantities, i.e. car and traffic density, mean occupancy, mean velocity, velocity scattering, acceleration noise and their dependences in a flow. So, the number of models increases especially in the last decade classified in macroscopic, microscopic and mesoscopic (esp. kinetic) categories following the ideas of physics [2–4].

Macroscopic models try to construct time propagation equations for the measured mean values in analogy to continuum mechanics [3,5]. These models can be used to examine dynamic properties of traffic flow like traffic jams, clustering, queue discharges and on- or off-ramp behavior. Microscopic models are based on the ideas of molecular dynamics or cellular automata and therefore need for solution computer simulations [6–8]. Especially in the first model type, detailed knowledge about the interaction behavior of drivers is needed and therefore also measured via car following experiments [9,10]. With microscopic modeling not only the dynamics of traffic flow but also traffic states in stochastic equilibrium are calculated. These calculations result in phase-diagram-like dependences between the traffic flow quantities. In particular the fundamental diagram, a dependence between traffic density and car density is very important for capacity planning of streets and so a lot of measurement and analysis is done there. Mesoscopic models belong to the class of stochastic methods. Here analog to gas kinetics they represent a link between micro- and macroscopic ones. They are mainly based on Boltzmann- or Enskog-like master equations for the single car state probability function [5,11] or for car cluster probability functions [12,13]. From this function, macroscopic flow quantities are calculated using mean values. In literature mesoscopic models are often used to derive macroscopic flow equations via moment or Chapman-Enskog methods [4,14,15]. There are only few papers, where those master equations are solved directly to reproduce the fundamental diagram [16–18] or traffic flow dynamics, e.g. traffic jams [19]. As in gas kinetics the master equations are of integro-differential type and difficult to solve numerically via discretization methods. So, in this paper a modified direct simulation Monte-Carlo (DSMC) method is introduced to solve a mesoscopic traffic flow model and to calculate

flow quantities in stochastic equilibrium.

In analogy to gas kinetics in most mesoscopic models the state space of a single car is given by the spatial and the velocity coordinate. The master equation approximates velocity changes due to interactions by unsteady jumps, assuming the Boltzmann limit, this means the time between interactions of a car is much larger than the interaction duration. This approximation holds true for dilute car densities only, as car following experiments show [9]. To relax this restriction, the author developed an enhanced model [20]. There the car acceleration is added to the state space as an independent variable in contrast to physics, where forces are mainly functions of the state space variables. Changes due to interactions now produce acceleration jumps. This approximation holds also for higher car densities, because the duration of an acceleration change is much smaller than any other traffic flow relevant time scale. The basic model equation of the time propagation of the state probability density is derived from the standard Chapman-Kolmogorov master equation assuming a vehicular chaos ansatz additionally. Especially for a spatial homogeneous traffic flow it reduces to a Boltzmann type equation. Before analyzing the full dynamic of a traffic flow, the homogeneous and stationary case must be investigated. In contrast to gas kinetics, where the stochastic equilibrium velocity distribution is universal due to the 'Stoßinvarianten', in traffic flow it depends on the interaction profile used. From measurements only a qualitative Gaussian shape distribution can be assumed. Therefore it is very important to study the equilibrium in velocity and acceleration for road capacity planning or for constructing closure conditions for macroscopic modeling.

In this paper the Boltzmann equation is solved for a special interaction profile numerically to study the stochastic equilibrium of a traffic flow and to compare the model solution, i.e. the car density dependence of the velocity and acceleration density together with their lower moments and the fundamental diagram to measured data. The numerical procedure is a modified DSMC method described in the next section in detail. Its convergence is shown for two simple, academic interaction cases, where analytical solutions are known.

2 Description of the Model and its Numerical Representation

The following section presents the traffic flow model shortly and shows its transition into the DSMC framework. A more detailed description of the model findings is published in [21,22].

2.1 The Homogeneous Model Equation

In the developed model all traffic flow features are calculated from the single car state probability density f , where $f(x, v, a, t)dx dv da$ is the probability of finding a car between x and $x + dx$ with velocity between v and $v + dv$ and acceleration value between a and $a + da$. As mentioned above, it is calculated from a modified master equation. For a spatial homogeneous traffic flow, the time propagation of the x -independent function $f(v, a, t)$ then reduces the basic master equation into a Boltzmann type one

$$\frac{\partial f}{\partial t} + a \frac{\partial f}{\partial v} = \int_{\bar{v}, \bar{a}, a'} (\Sigma(a | v, a', \bar{v}, \bar{a}) f(a', v, t) - \Sigma(a' | v, a, \bar{v}, \bar{a}) f(a, v, t)) f(\bar{a}, \bar{v}, t) d\bar{v} d\bar{a} da' . \quad (1)$$

Here (\bar{v}, \bar{a}) is the kinematic state of the leading car and Σ is the weighted interaction rate function of the process. The left part of the equation is the total change of f in time, which equals the total change rate on the right side. Changes only occur due to interactions, producing jumps in the acceleration value of the following car. Assuming that the duration of such an acceleration change is small against all other process intrinsic timescales, the total change rate can be expressed by the Boltzmann-like interaction integral mentioned in eq. 1. The weighted interaction rate function

$$\Sigma(a | a', v, \bar{v}, \bar{a}) = \int_{h \geq h_{\min}} Q(h, v, a', \bar{v}, \bar{a}) \sigma(a | h, v, a', \bar{v}, \bar{a}) D(h | v, a', \mathbf{m}_f) dh \quad (2)$$

is a compound of the interaction rate Q , the acceleration change probability density σ and the distance correlation function D . h_{\min} is the minimal distance between two cars at rest, i.e. the typical length of a car approximately. The rate $Q(h, v, a', \bar{v}, \bar{a})$ depends on a given microscopic state of a leading car pair $(v, a', \bar{v}, \bar{a})$ at a given distance h . $\sigma(a | h, v, a', \bar{v}, \bar{a}) da$ defines the probability of changing the acceleration of a following car to a value between a and $a + da$ under the condition that an interaction occurs and the leading car pair is in state $(v, a', \bar{v}, \bar{a})$ with distance h . The correlation function $D(h | v, a', \mathbf{m}_f)$ is equal to the conditioned distance probability density. It only depends on the state of the following car in a leading car pair, because the state of the leading car (\bar{v}, \bar{a}) is hard to determine for the driver even qualitatively. In addition the distance behavior depends on the whole traffic flow, which in the model is determined by the probability density f . Naturally, driver cannot observe f itself, but some lower order moments of f like the mean velocity, the mean acceleration and maybe the acceleration or velocity scattering etc.. Therefore D depends also on such a moment vector \mathbf{m}_f of f piggybacking the car density K as a parameter.

Beside the concrete specification of the interaction, initial and boundary conditions must be specified. The boundary conditions must ensure that no car can have negative velocities, $f(v = 0, a < 0, t) = 0$. Also, there exists a maximal free flow velocity w , which cannot be exceeded by the cars, $f(v = w, a > 0, t) = 0$. A more detailed discussion of the boundary conditions can be found in [20]. The stochastic equilibrium does not depend on an initial condition and the way how it is reached. Therefore as initial a normal velocity density without any acceleration is used, i.e.

$$f(v, a, t = 0) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-\frac{(v-V_0)^2}{2\sigma_0^2}} \delta(a) , \quad (3)$$

where $\delta(x)$ is the standard Dirac function. The parameters V_0 and σ_0 are chosen in such a way that the initial density $f(v, a, t = 0)$ lies near the resultant equilibrium density $f(v, a)$ to minimize the computational effort per run.

2.2 Numerical Solving Procedure

The Boltzmann type equation 1 is solved numerically for concrete interaction profiles defined below using a DSMC method well established in non equilibrium gas kinetics [23,24]. There, the Boltzmann equation is simulated by splitting the movement and the interaction (collision) process in two parts executed consecutively in a time interval Δt smaller than any characteristic timescale of the system. In the framework of the homogeneous model described here the splitting is done between the acceleration oriented interaction and the continuous velocity change [25]. Time integration of eq. 1 over Δt results in

$$\begin{aligned} f(v, a, t + \Delta t) &= (1 - \mathcal{D}\Delta t + \mathcal{J}\Delta t)f(v, a, t) + \mathcal{O}(\Delta t) \\ &= (1 - \mathcal{D}\Delta t)(1 + \mathcal{J}\Delta t)f(v, a, t) + \mathcal{O}(\Delta t) , \end{aligned} \quad (4)$$

up to first order in Δt , i.e. $\mathcal{O}(\Delta t)$. $\mathcal{D} = a\partial/\partial v$ is the velocity drift operator on the left side of eq. 1 and \mathcal{J} is the nonlinear operator on the right side of eq. 1 describing the un-continuous, stochastic acceleration change. The second equal sign in eq. 4 shows the serialization of the two processes. First, in a given small time interval Δt , $f(v, a, t)$ is changed in the acceleration variable resulting in a new, intermediate probability density $f_I(v, a, t) = (1 + \mathcal{J}\Delta t)f(v, a, t)$. Then this intermediate probability density is changed in the velocity due to the drift operator $(1 - \mathcal{D}\Delta t)$ into the new state probability density at $t + \Delta t$. Beside this serialization ansatz $f(v, a, t)$ is approximated by a discrete N -point measure

$$f(v, a, t) = \frac{1}{N} \sum_{i=1}^N \delta(v - v_i(t))\delta(a - a_i(t)) . \quad (5)$$

This measure can be interpreted as a number of N stochastic cars. Note that a stochastic car may not be confound with a real microscopic car, because it has no realistic velocity-time trajectory. This is the same difference known between a microscopic molecular dynamic picture and the Boltzmann one in gas kinetics.

Using eq. 5 and the definition of \mathcal{J} , the intermediate probability density f_I is calculated as

$$\begin{aligned} f_I(v, a, t) &= (1 + \mathcal{J}\Delta t)f((v, a, t)) \\ &= \frac{1}{N} \sum_{i=1}^N \delta(v - v_i) \{P_i p(a|v_i, a_i) + \delta(a - a_i)(1 - P_i)\} \end{aligned} \quad (6)$$

suppressing the time dependence of v_i and a_i for simplicity. Here

$$P_i = \frac{\Delta t}{N} \sum_{j=1}^N \int_a \Sigma(a|v_i, a_i, v_j, a_j) da = \frac{\Delta t}{N} \sum_{j=1}^N \Sigma_t(v_i, a_i, v_j, a_j) = \sum_{j=1}^N P_{ij} \quad (7)$$

is the probability that the i -th car interacts, P_{ij} is the probability that car i interacts with the leading car j and

$$p(a|v_i, a_i) = \frac{\Delta t}{N P_i} \sum_{j=1}^N \Sigma(a|v_i, a_i, v_j, a_j) = \sum_{j=1}^N \frac{\Sigma(a|v_i, a_i, v_j, a_j)}{\Sigma_t(v_i, a_i, v_j, a_j)} \cdot \frac{P_{ij}}{P_i}. \quad (8)$$

Eq. 8 consists out of a sum of two factors. For a given car i , the factor P_{ij}/P_i is the probability that car j is its interaction partner, i.e. the leading car. For a given leading car pair (i, j) which interacts, the new acceleration value of car i is taken from the probability density Σ/Σ_t , which is the other factor.

After the first step of the serialization procedure all N stochastic cars have the same velocities as before but some of them have new acceleration values. The intermediate probability density must be also a discrete measure

$$f_I(v, a, t) = \frac{1}{N} \sum_{i=1}^N \delta(v - v_i(t)) \delta(a - a_i^*(t)). \quad (9)$$

Here $a_i^* = a_i$ with probability $1 - P_i$ or has a new value $a_i^* \neq a_i$, sampled from P_{ij}/P_i and Σ/Σ_t with probability P_i . The second step of the serialization procedure is drifting the velocities of all N cars over Δt , leading in the new state probability density at $t + \Delta t$. This is done operating the drift operator on eq. 9 resulting

$$\begin{aligned}
f(v, a, t + \Delta t) &= (1 - \mathcal{D}\Delta t)f_I(v, a, t) \\
&= \frac{1}{N} \sum_{i=1}^N \delta(a - a_i^*) \underbrace{\left(\delta(v - v_i) - a_i^* \Delta t \frac{d}{dv} \delta(v - v_i) \right)}_{= \delta(v - (v_i + a_i^* \Delta t)) + \mathcal{O}(\Delta t)} \\
&= f_I(v - a\Delta t, a, t).
\end{aligned} \tag{10}$$

Setting $a_i(t + \Delta t) = a_i^*(t)$ and $v_i(t + \Delta t) = v_i(t) + a_i^*(t) \cdot \Delta t$ leaves eq. 10 in the form of eq. 5 at $t + \Delta t$. This kind of simulating the interaction part of eq. 1 first mentioned by Nanbu [24], although developed for gas kinetics is not frequently used there. The main argument against this procedure is the lack of energy and momentum conservation in a single interaction, which increases the scattering of the results. Therefore in gas kinetics the alternative Bird procedure [23] is mainly used. In traffic flow no conservation laws in an interaction must be considered, so the procedure described above seems to be the right choice. Its direct implementation leads to a high N^2 -computational effort, because of the calculation of the sum eq. 7 for each car $i = 1 \dots N$. This can be avoided using a special sampling procedure well established in literature [26,27]. First the possible interaction pair is calculated from P_{ij} and then it is decided, whether the interaction occurs really. This procedure restricts the integration time step to $\Delta t \leq 1/\Sigma_t$ giving rise to a time step control. Result sampling is done from the discrete states of all stochastic cars at given time points using histogram methods or standard mean value calculations. The stochastic equilibrium is assumed to be reached, if changes in the result quantities are ascertained to be small enough. The described simulation procedure is summarized in an algorithmic manner as follows:

0. start
1. repeat for M times
2. sample all (v_i, a_i) from the initial condition eq. 3, restricted on the $4\sigma_0$ -interval $[V_0 - 4\sigma_0, V_0 + 4\sigma_0]$
3. repeat for each time step Δt until t_{\max} is reached
4. repeat for each of the $i = 1, \dots, N$ cars
5. calculate uniform random number $R \in [0, 1]$
6. find leading car j from $(j - 1)/N \leq R < j/N$
7. calculate P_{ij} from eq. 7
8. if $P_{ij} > j/N$ then
9. scale down step size, $\Delta t/2 \rightarrow \Delta t$
10. reset all calculations of this time step
11. go back to step 4.
12. else
13. if $j/N - P_{ij} < R$ then
14. sample new acceleration a_i^* of the i -th car from Σ/Σ_t
15. else
16. do not change acceleration $a_i \rightarrow a_i^*$

17. next car i
18. repeat for each of the $i = 1, \dots, N$ cars
19. $v_i + a_i^* \Delta t \rightarrow v_i, \quad a_i^* \rightarrow a_i$
20. check boundary conditions at $v = 0$ and $v = w$ for car i
21. next car i
22. enlarge step size by 10%, $1.1 \Delta t \rightarrow \Delta t$
23. if simulation time reaches result sampling time,
then sample results
24. next time step
25. next run
26. output histograms and sample means of the M runs
27. stop

For calculating random numbers out of given distributions or for making decisions based on probabilities, the well known acceptance/rejection method is applied. A standard 32 bit linear congruential random number generator is used to produce uniform random numbers in the $[0, 1]$ -interval, because it possesses the lowest number of operations for their production. The influence of the random number quality is tested, by replacing the linear generator by a GFSR-type one [28], showing no significant differences in the results. This outcome can be justified due to the lack of tuples of consecutive random numbers for the same quantity in the algorithm, where congruential generators are known to be not adaptable.

The algorithm is implemented in FORTRAN 95 [29] and executed on a standard LINUX-PC with different initial parameters. Several thousand runs with more than one thousand stochastic cars are tractable with good response times.

3 Application of the Simulation Procedure

In this section results of the model using the simulation procedure described above are presented. At first, the applicability of the procedure is tested for a simple interaction case, where analytical solutions are known for comparison. Then a more realistic interaction profile is applied and a comparison to measured data is done.

3.1 Model Cases Compared with Analytical Solutions

In the following a spatial homogeneous, nearly free, stationary traffic flow, where all cars have velocities around the mean velocity $V \gg 0$ is consid-

ered. In each leading car pair for distances large enough measurements show a strong relative velocity dependence of the interaction. So the rate Q can be approximated by a function of the relative velocity $v_r = \bar{v} - v$ only. Assuming no interaction for equal velocities $v_r = 0$, Q can be Taylor expanded up to first order in v_r resulting [20]

$$Q(h, v, a', \bar{v}, \bar{a}) = Q(v_r) = r_0 \cdot |\bar{v} - v|, \quad \text{with} \quad r_0 = \left. \frac{dQ}{dv_r} \right|_{v_r=0}. \quad (11)$$

In this nearly free traffic flow interactions produce only small correcting changes in the acceleration value of the following car. This is approximated by a single value acceleration a_0 change probability density [20]

$$\sigma(a|h, v, a', \bar{v}, \bar{a}) = \sigma(a|v_r) = \begin{cases} \delta(a - a_0), & \text{if } v \leq \bar{v} \\ \delta(a + a_0), & \text{if } v > \bar{v}, \end{cases} \quad (12)$$

depending on whether the following car is faster or slower than the leading one. Inserting the interaction profile eqs. 11, 12 into eq. 2 leads to

$$\Sigma(a|a', v, \bar{v}, \bar{a}) = \Sigma(a|v_r) = r_0 |\bar{v} - v| \cdot \begin{cases} \delta(a - a_0), & \text{if } v \leq \bar{v} \\ \delta(a + a_0), & \text{if } v > \bar{v}, \end{cases} \quad (13)$$

with $\Sigma_t(a, v, \bar{v}, \bar{a}) = r_0 |\bar{v} - v|$. Under these conditions, eq. 1 can be solved analytically in stochastic equilibrium [20], resulting in

$$f(v, a) = \frac{1}{\sqrt{2\pi}\sigma_v} e^{-\frac{(v-V)^2}{2\sigma_v^2}} \cdot \frac{\delta(a - a_0) + \delta(a + a_0)}{2}, \quad (14)$$

with mean V and variance $\sigma_v^2 = a_0/r_0$. This solution is compared to the results of the computer simulation with $N = 1000$ stochastic cars and $M = 500$ runs and shows excellent agreement as can be seen in fig. 1a.

Changing the interaction rate Q from a relative velocity behavior, eq. 11, to a constant time threshold T , i.e. $Q = 1/T$, which corresponds to a change from a hard sphere interaction to a Maxwell one in gas kinetics, the solution in stochastic equilibrium is given by [20]

$$f(v, a) = \frac{\pi}{4\sqrt{3}\sigma_v \cosh^2\left(\frac{\pi}{2\sqrt{3}} \frac{v-V}{\sigma_v}\right)} \cdot \frac{\delta(a - a_0) + \delta(a + a_0)}{2} ., \quad (15)$$

with mean V and variance $\sigma_v^2 = (\pi T a_0)^2/3$. In fig. 1b this solution is compared to computer simulation results using the same parameter values, showing also excellent agreement. So, both model cases validate the applicability of the described algorithm to the model master equation.

Note that in both cases the mean velocity is a free parameter without a car density dependence. This comes from the lack of distance dependence in Q and σ , which let the weighted interaction rate function Σ independent of the distance correlation D .

3.2 A Realistic Interaction Profile Comparing with Measured Data

In this subsection the computer simulation method is applied to the model equation with a more realistic interaction profile. With this profile a comparison to measured data is possible. It is described roughly as far as it is relevant for the computer simulation method. Details are shown in [22].

A lot of microscopic car following models and their comparison to driver behavior show that the interaction between a leading car pair is distance threshold oriented at medium and high car densities with a threshold function mainly dependent on the velocity of the following car [30]. The interaction strength seems to be roughly proportional to the relative velocity $\bar{v} - v$ and inverse proportional to the car pair distance equal to the threshold $H_2(v)$ [31,32]. Interaction profiles have to be modeled to avoid accidents. Because relative velocity dependent acceleration values are not sufficient to ensure this, a second, lower value distance threshold $H_1(v) < H_2(v)$ is introduced. There, the de acceleration in the case of a decreasing distance is equal to the maximum braking value $-v^2/(2H_1)$ and in the case of an increasing distance is set to zero. From these considerations the conditioned change probability density $\sigma(a|h, v, a', \bar{v}, \bar{a})$ can be modeled on the thresholds H_1 and H_2 . The interaction rate Q for such a double threshold can be written as [17,22]

$$Q(h, v, a', \bar{v}, \bar{a}) = \sum_{l=1}^2 |v - \bar{v} + H'_l(v)a'| \delta(h - H_l(v)) , \quad (16)$$

where $H'_l(v)$ is the first derivative of H_l . $H_1(v)$ is set to a linear function and $H_2(v)$ is set to a square root like, sub linear one ensuring always $H_2(v) > H_1(v)$. The distance correlation function D must also only be known on the thresholds H_l . On each H_l there are two possible values $D(H_l|a', \mathbf{m}_f)$ depending on whether the distance increases or decreases. These values are approximated by the standard gamma probability density, where always a different weight factor is taken into account. The parameters of the gamma density are modeled to depend on the car density K and the mean velocity V of the flow

in stochastic equilibrium, i.e. $\mathbf{m}_f = (K, V)$. From the three functions σ , Q and D , the weighted interaction rate function eq. 2 is calculated. If an interaction between cars of a leading car pair occurs, the new acceleration value is calculated from the probability density

$$\frac{\Sigma(a|v, a', \bar{v}, \bar{a})}{\Sigma_t(v, a', \bar{v}, \bar{a})} = P_T \sigma(a|H_1, v, a', \bar{v}, \bar{a}) + (1 - P_T) \sigma(a|H_2, v, a', \bar{v}, \bar{a}), \quad (17)$$

where

$$P_T = \frac{|\bar{v} - v - H'_1(v) \cdot a'| D(H_1|a', K, V)}{\sum_{l=1}^2 |\bar{v} - v - H'_l(v) \cdot a'| D(H_l|a', K, V)} \quad (18)$$

is the probability of having an interaction on threshold H_1 and $1 - P_T$ on threshold H_2 . In the computer simulation algorithm for each interacting leading car pair (i, j) , step 14. is split into two parts. First, the threshold l is sampled using eq. 18 with the acceptance/rejection method and second, the new acceleration value is sampled using eq. 17.

The following figures show some results of the simulation together with measured data out of literature [33,34]. A detailed discussion of these and additional results can be found in [22]. Fig. 2 shows the car density dependence of the mean velocity V and the traffic density $q = K \cdot V$. Both quantities are scaled to each free flow velocity, because they are different in the measurement and in the simulation. Fig. 3 compares velocity distributions for different car density values and fig. 4 shows the car density dependence of the acceleration scattering often called acceleration noise (ACN).

4 Discussion of the Results

In this section measured data and the simulation results are compared and discrepancies are discussed. All figures show a qualitative agreement between the simulation results and the measured data. Where at medium and high car densities, i.e. $K \gtrsim 0.04/\text{m}$, the agreement in figs. 2 and 4 is significantly better, at low car densities, i.e. $0.01/\text{m} \lesssim K \lesssim 0.04/\text{m}$, there are qualitative discrepancies. In this density region the transition from free flow to interaction oriented flow occurs. In the model the free flow is rendered by a simple boundary condition introducing a maximal velocity w for all cars. Therefore a strong change from free flow with mean $V = w$ to a flow with distance oriented interactions $V < w$ appears. The measurement shows a less strong change of the mean velocity, fig. 2, in the low car density area of the transition regime, which can have several possible explanations:

- (i) Car following experiments show that for larger distances possible interactions are not distance oriented any longer, but relative velocity dependent [31]. Such a relative velocity interaction is not included in the interaction model described in 3.2. Because interactions change the acceleration value and so indirect the velocity value of cars, the mean value V must be slightly smaller than the free flow mean w .
- (ii) Measurements show that the free flow regime depends on multiple lane effects up to now not included into the model [35].
- (iii) In the model all car/driver pairs behave similar using the same interaction profile. In contrast to this in a real traffic flow there are a lot of different behaving car/driver pairs, which can be pooled into different car/driver classes. Where at high car densities the class effect vanishes because of an *enslavement* process by the slowest class, it is important in the free flow and the transition regime reducing the mean velocity. This effect is measured in a car free flow with a small percentage of trucks [5]. Further work to involve multiple classes into the model is under way.

The decrease effect of the mean velocity at low car densities produces a shift of the maximum of the measured traffic density in the fundamental diagram to higher K -values. The height of the traffic density maximum is approximately equal to that of the computer simulation. Therefore the simulation reproduce the maximum street capacity in the right order of magnitude.

The simulated velocity distributions lie in the same velocity area than the measured ones. They show lower deviations than the data, especially at low car densities. Because this effect is strongest in the free flow, it seems to be influenced by the model boundary condition and the arguments mentioned above.

There are a lot of models in vehicular traffic flow describing car density dependences of velocity quantities. In contrast to these, acceleration behavior is modeled much more seldom. Here lies the strength of the model with low numeric scattering in addition. The ACN in fig. 4 shows a qualitative agreement between the computer simulation results and data taken from a car following experiment. Like in fig. 2 the comparison is much better at medium and high car densities than at low ones. In a free flow the model boundary condition generates no acceleration scattering, where in reality ACN occurs always, even if a single car without interactions is regarded. So there is a difference of about $0.2-0.3\text{m/s}^2$ at low K -values. In the transition regime there is an increasing number of interactions producing different acceleration values and a strong rising ACN-value. The area of steep rise in the data is shifted to higher car densities because of the effects mentioned above. The absolute value seems to be in the right order of magnitude.

Note that in contrast to physics the mathematical structure of the weighted interaction rate (cross section) is not the same for all interaction partners, because drivers decide individually or in classes. Nevertheless the comparison realized shows that a simple threshold oriented interaction seems to be able to render the main traffic flow features in stochastic equilibrium. Especially the influence of driver behavior to the flow features can be studied without problems, because the computer simulation method is fast enough to produce quantitative discussible results without high computational effort.

5 Conclusion

1. In this paper the direct simulation Monte Carlo (DSMC) method is applied to a mesoscopic vehicular traffic flow model based on a Boltzmann-like master equation. The method is developed directly from the master equation following the ideas of Nanbu and Babovsky resulting in a fast simulation scheme, which is realized on a standard PC even for complex driver interaction profiles. The scheme is applied to several interaction profiles for comparison. Results are shown in the stochastic equilibrium case.
2. The computer simulation is compared to two different interaction model cases, where analytical solutions for the velocity distribution exists. In both interaction model cases the simulation and the analytical solution agree very good by low computational effort and fast convergence.
3. A realistic interaction profile simulation based on two velocity dependent distance thresholds on which acceleration changes occur is compared to measured data in stochastic equilibrium. They agree qualitatively in the medium and high car density region, where interactions rule the traffic flow. At low car densities significant differences occur, because non interacting driver behavior rules the traffic flow, which is included by a simple boundary condition for into the model up to now. Additional work must be done to increase the accuracy of the model in this area.
4. The model together with the fast simulation method allows a high quality analysis of traffic flow quantities in stochastic equilibrium. Especially the influence of the driver behavior expressed in the interaction profiles of the model can be studied without high numerical effort.

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Figure Captions

- Fig. 1 Velocity state density for a relative velocity depended (a) and a constant time (b) interaction in standardized form. Histogram data is taken from computer simulation, curves are analytical results from the acceleration integrated eqs. 14, 15.
- Fig. 2 Car density dependence of the scaled mean velocity V (a) and the scaled traffic density q (b), compared to measured data [34].
- Fig. 3 Velocity probability densities for different car density values K . Measured relative frequency from [34] above, model density below.
- Fig. 4 Car density dependence of the ACN together with measured data from [33].

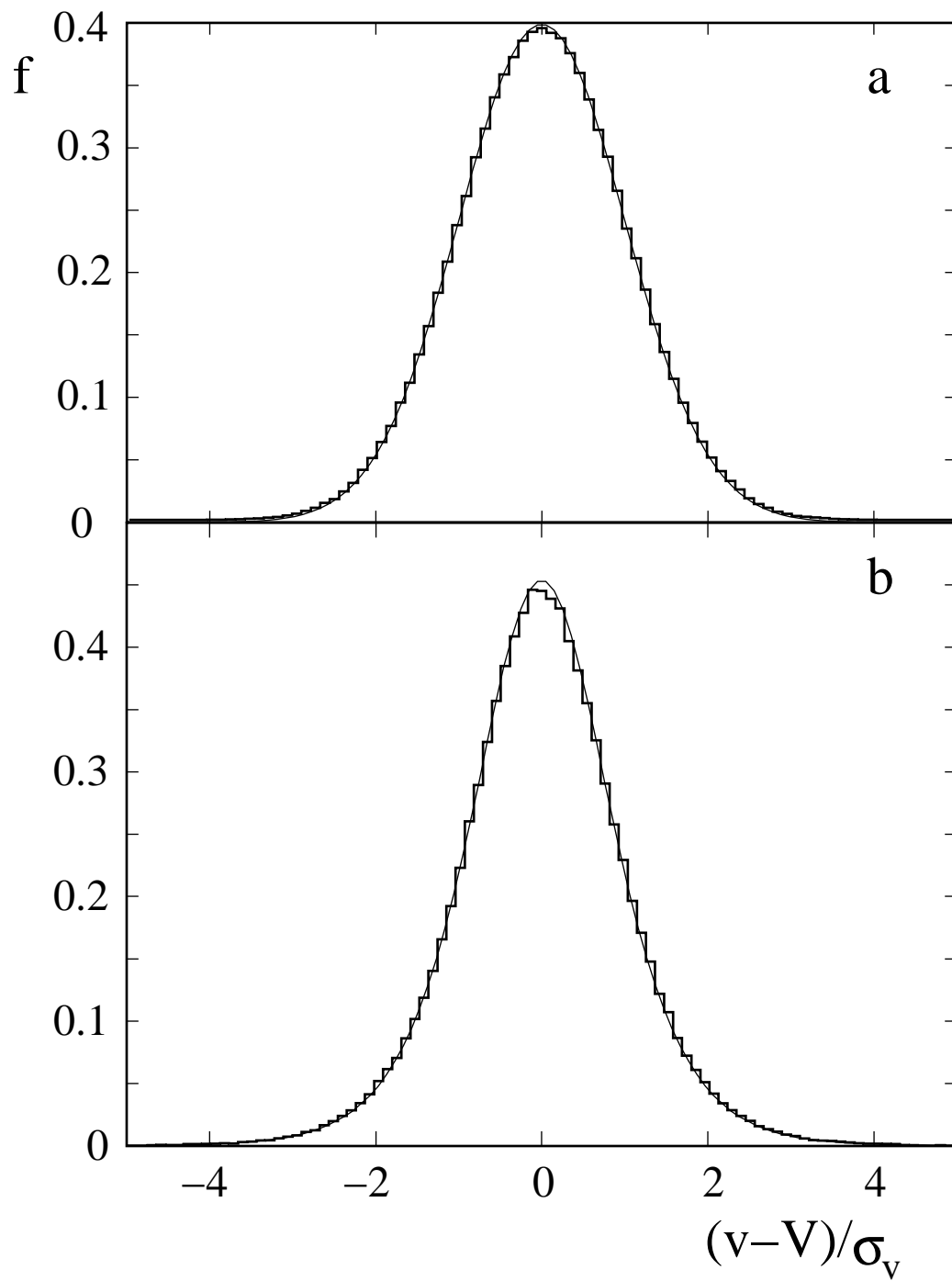


Fig. 1.

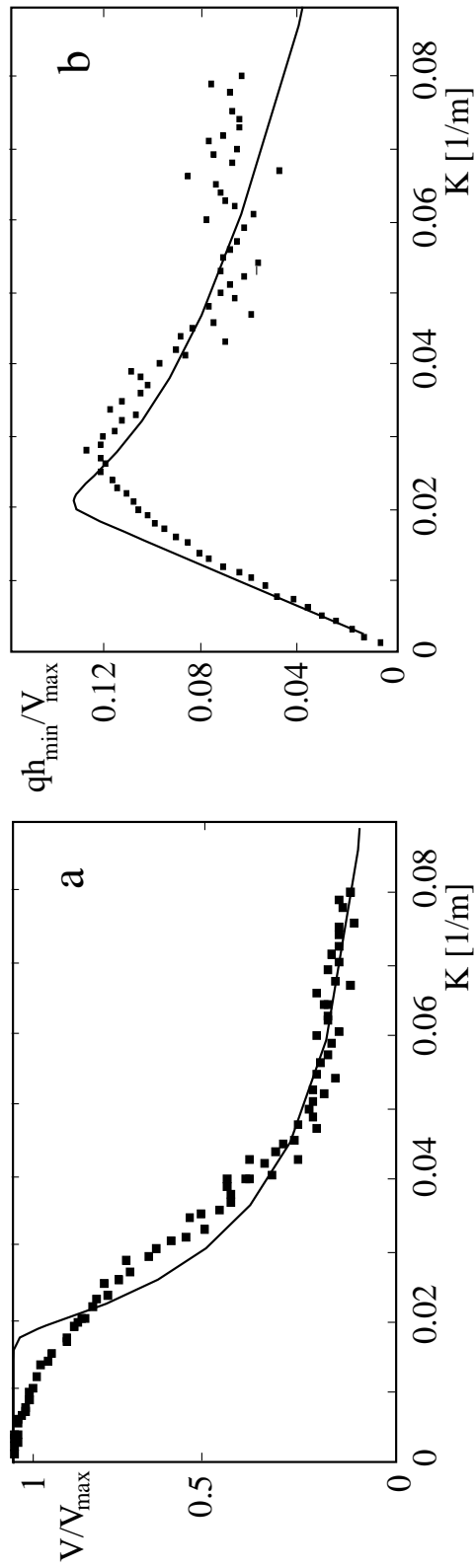


Fig. 2.

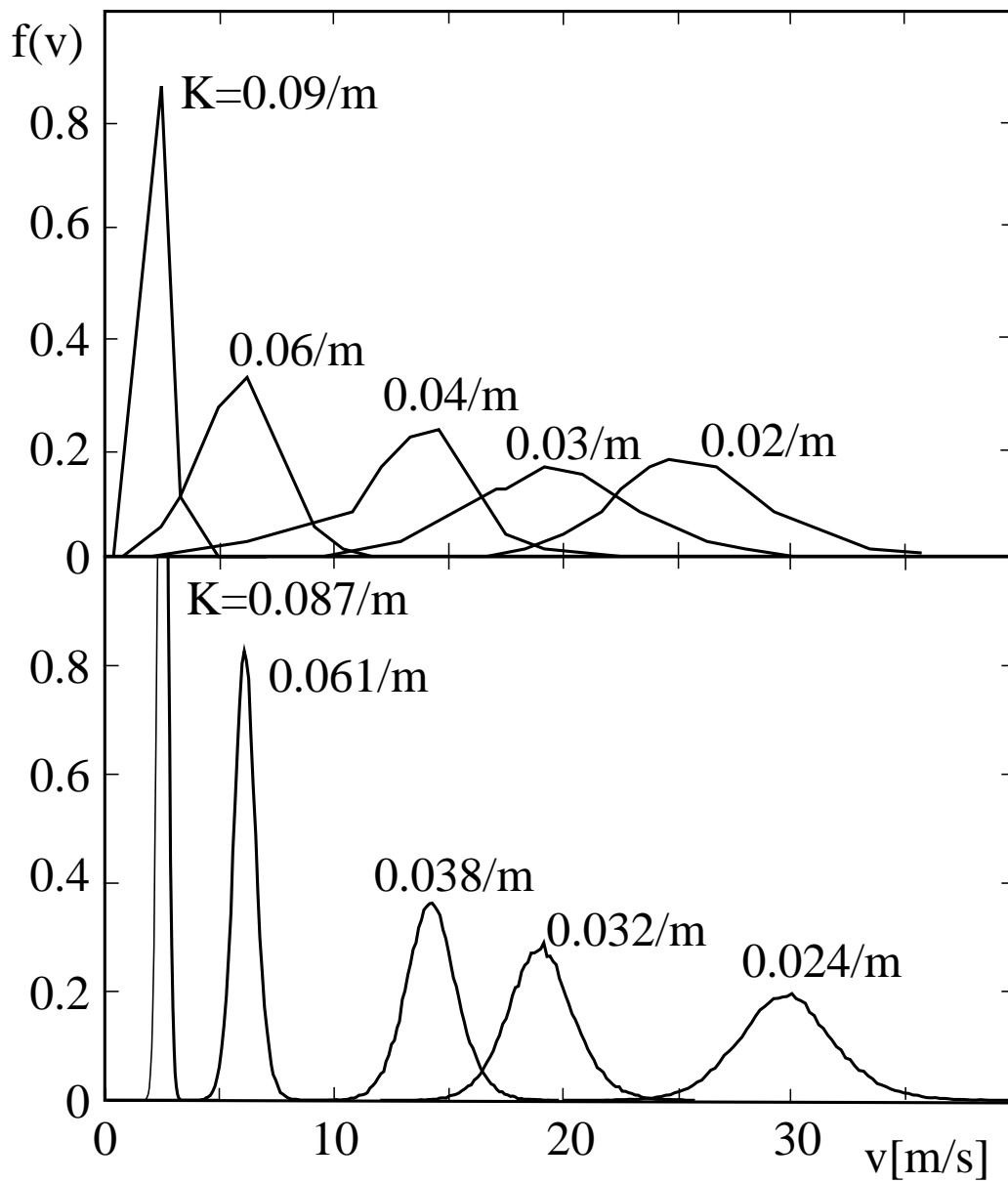


Fig. 3.

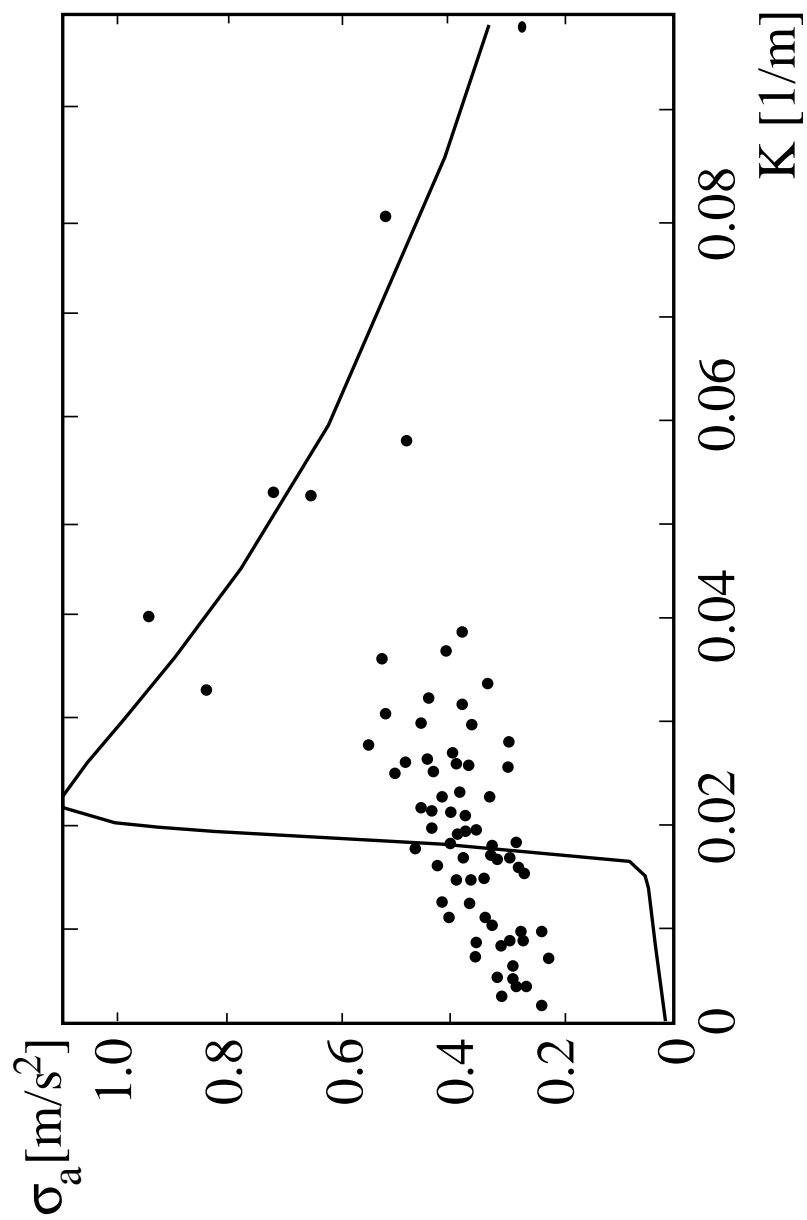


Fig. 4.