Systematics of the energy density of vacuum fluctuations and geometrodynamical excitones

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Abstract
In geometrodynamics, the energy density of the waves of quantum fluctuations leads to the well-known problem of energy- compensation within the explanation of the energy of geometro-dynamical excitones. The correlation between their energy density and the spatial dimension of their volume in connection with the modes of zero- point oscillations in the inside of their volume is the subject of the preceding article.

1. Introduction and basic idea

The topic of the energy density of the vacuum is a still unanswered question. On the one hand, from quantum electrodynamics and quantum field theory, a value of about $10^{+113}\,\frac{J}{m^3}$ is deduced from the vacuum fluctuation of geometrodynamics (see e.g. [WHE 68], [KÖP 97], [KUH 95], [FaS 49]). On the other hand, measurements from astrophysics and their interpretation on the basis of expansion of the universe leads to values of about $10^{-9}\,\frac{J}{m^3}$ (see e.g. [GIU 00], [TEG 02], [TUR 04a]). The discrepancy is more than 120 orders of magnitude.

The crucial question is now:
Is it possible to recognize a relation between these totally different values in order to explain them ? This connection can be found – it is presented on the following pages – together with first hints to fundamental explanations. A graphical illustration of this connection is given in fig.1. Because of the fact, that the several data descend from very different origin, they are explained prior to fig.1 in the chapters 2 to 7.

The basic idea of all presented calculations is the following:
Vacuum is space containing something, it is not nothing. Issue of our consideration is the energy density of this non empty space.
Mostly it is implicitely supposed, that the energy density shall be constant as a function of the spatial dimension. With other words: It is assumed, that the energy density of the vacuum is always the same – no matter how large the volume of the vacuum is, which is under consideration. The central idea of this work here is to disclaim this presumption:

Let the energy density of the vacuum allow to depend on the size of the analysed structure in space.

The plausibility of this postulate is obvious since H.B.G.Casimir introduced the Casimir- effect [CAS 48], which has following background: If a limited volume is insulated from the rest of the vacuum, what Casimir did by the use of two perfectly conducting and electrically neutral plates, within this insulated volume only several special modes of the quantum electrodynamic vacuum fluctuations can exist. This restriction of the modes of oscillation does not occur within the free vacuum. Spatial limitation restricts the number of the allowed modes of oscillation.

Casimir calculated the vacuum's energy density within the insulated volume and further developed his calculations to obtain the force between the plates. This force can only be measured for very small distances between the two plates (see e.g. [LAM 97], [BRE 02], [MOH 98]), which can be understood because of the way how the energy density ρ decreases with increasing distance r of the plates, namely by the proportionality of $\rho \propto 1/r^4$.

If we make ourselves free from the way how Casimir insulated the little volume from the vacuum, and if we expand our considerations to other microscopically small volumes, insulated from the vacuum (for instance like elementary particles), we find out that their energy density fits well into Casimir's calculation. Surprisingly we even see that Casimir's calculation can be extrapolated to particles like electrons or protons and even down to the fundamental zero- point vibrations within vacuum on the length- scale of the Planck length.

Of course this can be understood as a hint onto the cause of the energy (and rest mass) of the insulated volumes: They could have their cause within the energy of those special geometro-dynamical vacuum fluctuations, which can exist within the insulated volumes, different from those vacuum fluctuations within the free unlimited vacuum (like in the universe). This can be understood as following:
The number of vacuum fluctuations in the free (unlimited) vacuum is tremendously large, containing frequencies nearly up to infinity. According to quantum electrodynamics, the frequencies could go to infinity, but because of geometrodynamics the wavelength has a lower limit at the Planck length. This produces an upper limit of the frequency.
In the inside of the elementary particles only those vacuum fluctuations can exist, whose wavelengths are short enough to fit into the volume of the elementary particle. This upper limit of the wavelength has the consequence of a lower limit in the frequency, depending on the size of the elementary particle. In contrast to the free vacuum, some long wavelengths are missing within the volume of the elementary particles. And exactly those missing waves define the energy of the particle.
By principle this is analogous to the Casimir- energy in insulated space between the Casimir-plates. The smaller the volume, the more wavelengths are missing (compared to the free vacuum), the higher the energy and the energy density of the volume. This correlation is analysed quantitatively on the following pages by the use of several different well- known data.

2. Casimir- effect

Let us regard two perfectly conducting and electrically neutral plates, each one with a surface area of L^2, oriented parallel to each other with a small distance **a**. For this arrangement, in [CAS 48] H.B.G.Casimir calculated the energy difference δE between the vacuum within the space between the plates and the free vacuum from the quantum electrodynamic zero- point oscillations as following

$$\delta E = \left(\tfrac{1}{2}\sum \hbar\omega\right)_I - \left(\tfrac{1}{2}\sum \hbar\omega\right)_{II} = \hbar c \frac{L^2}{\pi^2}\cdot\frac{\pi}{2}\left\{ \sum_{n=(0)1}^{\infty} \int_0^{\infty} \sqrt{\left(n^2 \tfrac{\pi^2}{a^2} + \kappa^2\right)}\kappa\, d\kappa - \int_0^{\infty}\int_0^{\infty} \sqrt{\left(k_z^2 + \kappa^2\right)}\kappa\, d\kappa \left(\tfrac{a}{\pi} dk_z\right)\right\} \tag{01}$$

with the plates being orientated parallel to the xy-plane, and with wave numbers of the zero-point oscillations as

$$k_x = \frac{\pi}{L} n_x,\ k_y = \frac{\pi}{L} n_y,\ k_z = \frac{\pi}{a} n_z, \qquad \text{where } n_x, n_y, n_z \in \aleph,$$

and $$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{\kappa^2 + k_z^2} \qquad \text{(with the use of polar coordinates),}$$

where the summation index n=(0)1 indicates, that the term with n=0 has to be multiplied with $\frac{1}{2}$.

By the means of appropriate mathematical operations, Casimir solves equation (01) and comes finally to an energy difference per surface area of the plates of:

$$\frac{\delta E}{L^2} = \frac{-\pi^2 \hbar c}{720 a^3} \tag{02}$$

The minus sign indicates, that the energy is missing compared to the free vacuum outside. The energy density ρ_C is the energy difference per volume, with $L^2 a$ being the volume between the plates, and thus its absolute value is

$$\rho_C = \frac{\delta E}{L^2 a} = \frac{\pi^2 \hbar c}{720 a^4}. \tag{03}$$

Please keep in mind, that the energy density ρ_C is proportional to a^{-4}.

The result of equation (03) is plotted in fig.1 as a line, marked with "C" (like Casimir). The length of the line follows the experimental verification of the Casimir effect, beginning at 20 nm [EDE 00] to increasing distances [MOH 98] and [BRE 02] up to about 6 µm [LAM 97].

3. Prediction of Geometrodynamics

In geometrodynamics, elementary particles are regarded as collective excitations of the geometrical continuum (the latter one as a synonym for the vacuum) and are thus called quantum geometrodynamical excitones (see e.g. [WHE 68]).

This corresponds to the description of elementary particles as insulated volumes as specified in chapter 1. Hence we face the question about the existence of a smallest possible volume – this would have to be the volume with the highest possible energy density. In classical electro-dynamics there is no such lower limit to geometry, but in quantum geometrodynamics there is such a limit: Planck's length, which is

$$L^* = \sqrt{\frac{\hbar G}{c^3}} = 1.6\cdot 10^{-35} m \tag{04}$$

Following standard geometrodynamics, for the associated zero- point oscillations let us assume Gaussian probability distribution, so that we receive

$$\Psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}x^2}$$

The energy of the vacuum oscillations now has to be calculated as a sum of the kinetic and potential energy as

$$E = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} = \int \Psi^*(x)\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega^2 x^2}{2}\right]\Psi(x)dx$$

which leads to an associated energy density of the smallest possible volume in quantum geometrodynamics of

$$\rho_G \approx \frac{\hbar c}{L^{*4}} = 4.6 \cdot 10^{+113}\, \tfrac{J}{m^3} \qquad (05)$$

This energy density is enormously large and aroused the already quoted problem of energy compensation.

It should be annotated, that this energy density at the quantum geometrodynamical limit of the zero- point oscillations is the same as Casimir's energy density except for a factor of $\pi^2/720$. The proportionality $\rho \propto 1/r^4$ is the same in both cases. Compared with the discrepancy of more than 120 orders of magnitude as mentioned above, this factor is almost negligible, especially if we keep in mind, that the quantum geometrodynamical value is a hypothetical estimation – other than Casimir's value which is verified experimentally.

The result of equation (05) is plotted in fig.1 and marked with "G" (like Geometrodynamics).

4. Light and electromagnetic waves

If we regard an electromagnetic wave as one single wave removed from the full spectrum of all the waves representing the vacuum's electromagnetic zero- point oscillations, we can insert its energy density into fig.1 as following:

The energy of an electromagnetic wave is common knowledge:

$$E = \frac{h \cdot c}{\lambda} \qquad (with\ \lambda = wavelength) \qquad (06)$$

But for the calculation of the energy density, we should have a volume to refer to. If all items of this world can be understood on the basis of geometry (as postulated in geometrodynamics) than also the photon should have a volume.

It is plausible, that the photon's volume is connected to its wavelength. This argument is also supported by the effect of diffraction, where the geometrical dimension of the diffracting structure has a rather close relationship to the wavelength.

So let us call $n_x \cdot \lambda$ the length of the photon in direction of its propagation and $n_y \cdot \lambda$ respectively $n_z \cdot \lambda$ its wideness perpendicular to the direction of propagation. Then its volume will be $V = n_x \cdot n_y \cdot n_z \cdot \lambda^3$ (with $n_x, n_y, n_z \in \Re$ three small factors for instance 1...10). Than the energy density will follow from equation (06) to be

$$\rho_W = \frac{E}{V} = \frac{h \cdot c}{n_x n_y n_z \cdot \lambda^4} \qquad (07)$$

We see: Also electromangetic waves have an energy density proportional to λ^{-4}.
Under the assumption, that every electromagnetic wave is one single wave taken out of the zero-point oscillations, this would have the consequence, that every single waves of the zero- point oscillations would have to follow the $\rho \propto 1/r^4$ - systematics.

A constant factor remains unsolved – namely $n_x \cdot n_y \cdot n_z$. Therefore the entry "W" (for electromagnetic wave) in fig.1 is a punctuated area (plotted for the product $n_x \cdot n_y \cdot n_z = 5..500$).
If Casimir's energy density would equate with the energy density of electromagnetic waves (equations 03 and 07), this would bring us $n_x \cdot n_y \cdot n_z = 458$. If we compare the geometrodynamical limit with the energy density of electromagnetic waves (equations 05 and 07), we would come to $n_x \cdot n_y \cdot n_z = 2\pi$.

5. The electron

In order to regard the electron, we have to assign a volume to it, according to the needs of geometrodynamics. Together with the classical attitude, we might treat the electron as a charged sphere (see e.g. standard textbooks like ([BER 79] or [FLS 91]) and calculate its volume from its radius r_e (see e.g. [COD 00], [TUR 04b]).
So the calculation of the energy density of this sphere is a rather simple matter:

$$\rho_E = \frac{E}{V} = \frac{m_e c^2}{\frac{4}{3}\pi r_e^3} = 1.64 \cdot 10^{+23} \frac{J}{m^3} \ , \qquad (08)$$

and leads to the entry "E" (like electron) in fig.1. Also the electron fits well into the $\rho \propto 1/r^4$ - systematics of the insulated volumes.

6. nucleons (proton and neutron)

Different from leptons, nucleons are composed of quarks. Therefore we could discuss about the necessity to regard all individual quarks as insulated volumes, with the consequence that the proton could not be regarded as one single insulated volume. Contrary to this argument, we know, that single quarks can never be isolated, and so we refuse to treat them as isolated (or insulated) volumes.
Regarding our $\rho \propto 1/r^4$ - systematics we will soon see, that protons as well as neutrons can be treated as insulated volumes (obviously the quarks are kept so tight together by gluons, that zero- point oscillations from the free vacuum can not penetrate into the space between them).

Our calculation proceeds as following:
On the scale used in this article here, protons and neutrons are so similar to each other, that they have almost the same point in fig.1. So it is enough to do the calculation only for one of them. Let us choose (arbitrarily) the proton.

The calculation proceeds rather analogous to the calculation of the electron; mass and radius of the proton are common knowledge. The mass can be found for instance in [COD 00]. For the proton's radius r_p we know theoretical values from relativistic bag- theory (see e.g. [CHO 74]) of $1.0\,fm$ and experimental values of $0.86\,fm$ from [HAN 63] or $0.81\,fm$ from [SIM 80]. An intermediate values of $r_p = 0.9$ fm should be reasonable for further use. For the energy density we then receive

$$\rho_N = \frac{E}{V} = \frac{m_P c^2}{\frac{4}{3}\pi r_P^3} = 4.9 \cdot 10^{+34}\ \tfrac{J}{m^3} \qquad (09)$$

The result is plotted in fig.1, marked as "N" (like nucleon), and also fits into the $\rho \propto 1/r^4$ – systematics.

7. General considerations for elementary particles

After having verified our $\rho \propto 1/r^4$ – systematics for a lepton and two hadrons, the question arises, whether this systematics can be applied to all elementary particles.
Well – we can not find an easy or a universal answer just now, at least because we do not quantitative values for the quantum geometrodynamical volume of all elementary particles. But nevertheless we can apply one basic consideration on principle, namely as following.
The spatial resolution of accelerators increases with the energy of the colliding particles (normally given in the center-of-mass frame). What we now need is a fundamental physical connection between the energy of the particles and the spatial resolution. This means: Which energy is necessary to achieve a resolution to dissolve a given length (or volume) ?
Interestingly we will find that the energy density corresponding to this energy again follows our $\rho \propto 1/r^4$ – systematics.

Of course the answer only can be significant, if it is absolutely independent from the any technology or machine – but it is exactly such an answer we find:
According to Abbé (see e.g. [BOR 85]), the optical resolution of a microscope is determined from the position of the maxima of interference. Hence it can be concluded, that the smallest resolvable structure $d_{\min}$ is of about the same size as the wavelength ($d_{\min} \approx \lambda$). This is considered to be the same behaviour for light as for matter waves, so that the wavelength can be scaled in energy of the particles (see [HIL 96]):

$$E_{sp} = h \cdot \frac{c}{\lambda} \qquad \text{(with } E_{sp} = \text{particle's energy in the center-of-mass frame)}$$

The smallest dissolvable structure is then

$$d_{\min} = \frac{h \cdot c}{E_{sp}} \quad , \qquad (10)$$

which is indeed free from any attributes of machines.
This means: For the production of elementary particles down to a geometrical diameter of $d_{\min}$ (and radius $r_{\min} = \frac{1}{2} d_{\min}$), it is necessary to apply the energy E_{sp}, having an energy density ρ_p being calculated as

$$E_{sp} = \frac{h \cdot c}{d_{\min}} \quad \Rightarrow \quad \rho_p = \frac{E_{sp}}{V} = \frac{E_{sp}}{\frac{4}{3}\pi r_{\min}^3} = \frac{h \cdot c}{2 r_{\min} \cdot \frac{4}{3}\pi r_{\min}^3} \qquad (11)$$

(Remark: For the sake of simplicity, the particle's volume is assumed to be spherical.)

Equation (11) describes how much energy and energy density is contained within the volume of an elementary particle of the quantum geometrodynamical diameter $d_{\min}$. Indeed this also fits into our $\rho \propto 1/r^4 -$ systematics in accordance with the equations (08) and (09). Thus it may be that equation (11) is valid (under the presumption of the spherical shape of the particles) for rather many elementary particles.
Consequently we might plot a line into fig.1. But we do not have any information about the length of the line, an so a single point is enough to represent the consideration of chapter 8. The choice of this point follows [HIL 96] where the planning of a modern accelerator is given as a numerical example:

$$Energy \quad E_{sp} = 10TeV = 1.6 \cdot 10^{-6} J$$

==> $d_{\min} = 1.24 \cdot 10^{-19} m$ ==> $r_{\min} = 6.2 \cdot 10^{-20} m$

and $energy\ density \quad \rho_p = 1.6 \cdot 10^{+51} \frac{J}{m^3}$

The name of the point in fig.1 is "P" (like particles) and also fits into our $\rho \propto 1/r^4 -$ systematics.

8. Combined diagram of all single data

A graphical overview of all our results is given in fig. 1.
The connection between the energy density ρ of a volume insulated with respect to the vacuum and the spatial dimension r of the volume can be seen very clearly: $\rho \propto 1/r^4$

This relation is valid for very different items, so it should have a rather fundamental significance, which has to be explained by a feature which all these points have in common. There is only one such feature: They are fundamental insulated volumes, which take out some of the modes of the zero- point oscillations from the vacuum. Following [WHE 68], such volumes can be called quantum geometrodynamical excitones. Individual differences between these excitones should have their reason in the fact, which of the zero- point oscillations an exciton individually allows and which not.

The fact, that for the Casimir- effect the factor of proportionality is a bit different from the factor of proportionality of the other objects might have its reason within the dimensionality of the effects: The parallel metal- plates at the Casimir- effect are to be regarded as one-dimensional limitations of a volume, whereas all the other objects act as three-dimensional limitations within space.

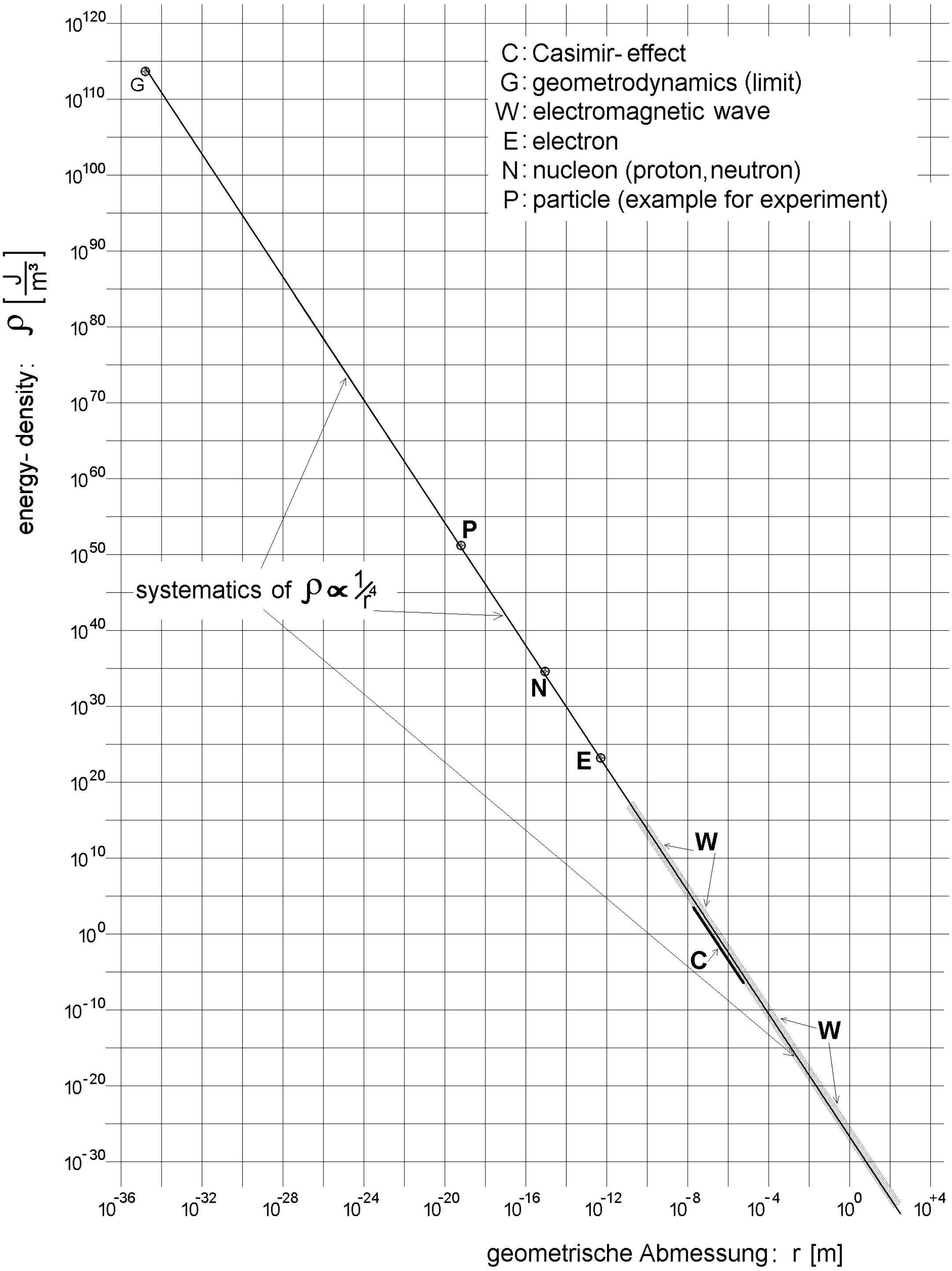

Fig.1.: Graphical presentation of the systematics of the energy density as a function of the spatial dimension of insulated volumina in the vacuum as quantumgeometrical excitonses because of the quantumgeometrical zero-point vibrations.

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