

About the Electrostatic field following Coulomb's law with additional Consideration of the finite speed of propagation following the Theory of Relativity

Wolfenbüttel, Dezember – 11 – 2007

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Abstract

The electrostatic field emanating from some electric charge can be calculated with the use of Coulomb's law. This law does not make any statement regarding the speed of propagation of the field, infinite speed is assumed automatically. But the Theory of Relativity does not allow such speed, it rather restricts the speed of propagation to the speed of light. If we take this into account, the electrostatic field strength of any charge depends also on the speed of propagation of the field. In this article the principle of the propagation of the electric field with finite speed is explained, and an example is demonstrated, in which the speed of propagation of the field is necessary to calculate electrostatic field strength.

Outline of the Article

- (1.) Explanation of the principle of Coulomb's law with additional consideration of the finite speed of propagation of the emanated field.
- (2.) An example of four oscillating point charges and their electrostatic field:
 - a. → their field strength under the assumption of **infinite** speed of propagation.
 - b. → their field strength under the assumption of **finite** speed of propagation.

Article body

(1.) Explanation of the principle of Coulomb's law with additional consideration of the finite speed of propagation of the emanated field.

Coulomb's law, as found in many textbooks even for students [1] gives the force between two point charges, which is often written as

$$\vec{F} = \frac{q_1 \cdot q_2 \cdot \vec{r}}{4\pi \epsilon_0 |\vec{r}|^3} \quad \text{with} \quad \begin{array}{l} q_1, q_2 = \text{electrical charges} \\ \vec{r} = \text{distance between both charges} \\ \epsilon_0 = 8.854187817 \cdot 10^{-12} \frac{\text{A}\cdot\text{s}}{\text{V}\cdot\text{m}} = \text{electrical field constant [2]} \end{array} \quad (1)$$

From there, we see rather often the conclusion to the field strength of charge no.1 as

$$\vec{E}(\vec{r}) = \frac{q_1 \cdot \vec{r}}{4\pi \epsilon_0 |\vec{r}|^3} \quad (2)$$

No thoughts are dedicated to the speed of propagation of the field, so that the field is assumed to have the given value at every position in the space instantaneously. The consequence would be, that an alteration of the electrical charge q_1 can be seen everywhere in the same moment, which would give us the possibility to transmit information with infinite speed [3] – in clear contradiction to the Theory of Relativity according to which the speed of propagation cannot exceed the speed of light.

In order to rid of the contradiction to the Theory of Relativity, we can treat the speed of propagation of the field in the calculation with a restriction to the speed of light, as it was done in [4]. One of the consequences would be that the field generated by an electric charge

(as for instance q_1 in (2)) will act with some certain delay, depending on the distance \bar{r} which the field has to pass on its way coming from the charge.

This point of view may appear a bit unusual, but it is just the point of view explaining the fundamental mechanism of the Hertz'ian dipole emitter for which in addition to the electrostatic field also the magnetic field has to be taken into account [5].

Another argument to accept this restriction of the propagation to the speed of light is the possible existence of gravitational waves. They can only exist, if the speed of propagation of the field of gravitation is restricted to the speed of light [6], [7], [8], [9]. If detectors of gravitational waves descending from oscillating masses are sensible, detectors of electrostatic waves, descending from oscillating electrical charges have to be the same sensible. We should remember, that Newton's law of gravitation, which has the dominant responsibility for the genesis of gravitational waves, has a form analogous to Coulomb's law. So, if the existence of gravitational waves is possible, the existence of electrostatic waves is possible in the same way.

This means, that an electrical charge will emanate electrical field, and this field propagates through the space with finite speed after it is emanated. The propagation of the field is not influenced by the position of the charge after the field is already emanated. So if the charge is moving during time, it emanates permanently, and the emanated field is coming from every position at the time the charge has been there.

This conception differs by principle from the conception of the infinite speed of propagation of the field, in which an alteration of the position of the charge would alter the field in the whole universe at the same moment – in contradiction to the Theory of Relativity.

(2.a.) An example of four oscillating point charges and their electrostatic field under the assumption of infinite speed of propagation.

As an example we want to choose two pairs of electrical point charges as seen in fig.2. But the principle explanation begins with fig.1, in which we look at two electrically charged spherical shells (we see their inner and outer border), which contract and expand periodically.

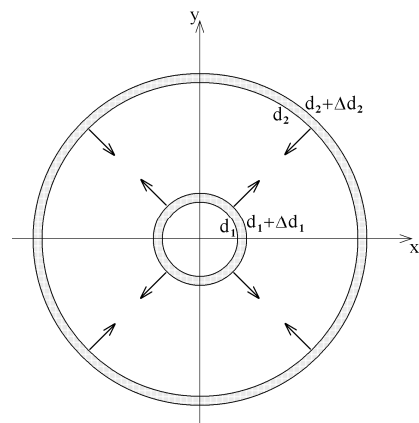


Fig. 1:

Illustration of two electrically charged spherical shells, contracting and expanding periodically. In the moment displayed here, the outer shell (no.2, carrying the charge $-q$) contracts and the inner shell (no.1, carrying the charge $+q$) expands until they will pass through each other, and finally until d_2 will be smaller than d_1 . The process of periodical contraction and expansion continues during all time of our observation.

The image shows a 2-dimensional projection of a 3-dimensional assembly of spherical shells, representing their borders.

If we follow Coulomb's law without taking time and the finite speed of propagation of the fields into account, each sphere in fig.1 will generate the same electrostatic field in the space outside the spheres, as if its charge would be concentrated in the point of its centre. Because the charges no.1 and no.2 are zero in sum ($-q + q = 0$) and have the same centre, the total fields of both charges sums up to zero, independently from the radii of the spheres and the alteration of these radii during time.

For the calculation taking the finite speed of propagation of the fields into account, as done in section 2.b, a 3-dimensional construction as shown in fig.1 is rather complicated, but a simplified 1-dimensional projection of this structure is already sufficient to analyze the field

and understand what happens. This is what we see in fig.2, where the positions of the charges along to x-axis (as one dimension taken from the sphere) can be seen as a function of time as well as the position of an observer not moving at all. In fig.2 the said positions along the x-axis of fig.1 are plotted versus the time for sinusoidal oscillations of the charges. For further easiness we allow the thickness of the spherical shells to be zero, this means $\Delta d \rightarrow 0$. Then the positive charges of one of the spheres with the enumeration $q_2^{(+)}$ and $q_3^{(+)}$ oscillates between the radii d_1 and d_2 as well as the negative charges of the other sphere with the enumeration $q_1^{(-)}$ and $q_4^{(-)}$ do. In fig.1 we see a snapshot representing the moment, where the positive charged sphere is smaller than the negative charged sphere. For the further calculation of our example let the positive charge have the same absolute value as the negative charge, and let us introduce the abbreviation q , i.e. $q := q_2 = q_3 = -q_1 = -q_4$.

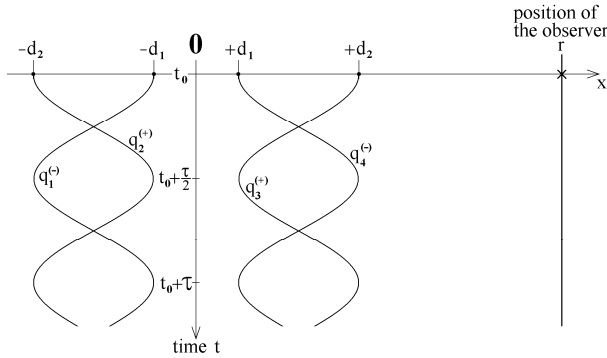


Fig. 2: Illustration of the oscillation of four charges during time as the given conditions for an example which will be calculated in the following lines as well as in section 2.b of the article.

After fig.1 and the above explanations it is clear, that also in our 1-dimensional example the sum of the fields of all four electrical charges (no. 1...4) is zero (along the geometrical dimension, which is the x-axis) – as long as the finite speed of propagation of the fields is not taken into account. This can be controlled with a simple calculation, showing that

$$\begin{aligned} \vec{E}_{\text{ges}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 &= \frac{q_1 \cdot (\vec{r} - \vec{s}_1)}{4\pi \epsilon_0 |\vec{r} - \vec{s}_1|^3} + \frac{q_2 \cdot (\vec{r} - \vec{s}_2)}{4\pi \epsilon_0 |\vec{r} - \vec{s}_2|^3} + \frac{q_3 \cdot (\vec{r} - \vec{s}_3)}{4\pi \epsilon_0 |\vec{r} - \vec{s}_3|^3} + \frac{q_4 \cdot (\vec{r} - \vec{s}_4)}{4\pi \epsilon_0 |\vec{r} - \vec{s}_4|^3} \\ &= \frac{-q \cdot (\vec{r} - \vec{s}_1)}{4\pi \epsilon_0 |\vec{r} - \vec{s}_1|^3} + \frac{q \cdot (\vec{r} - \vec{s}_2)}{4\pi \epsilon_0 |\vec{r} - \vec{s}_2|^3} + \frac{q \cdot (\vec{r} - \vec{s}_3)}{4\pi \epsilon_0 |\vec{r} - \vec{s}_3|^3} - \frac{q \cdot (\vec{r} - \vec{s}_4)}{4\pi \epsilon_0 |\vec{r} - \vec{s}_4|^3} = \vec{0} \end{aligned} \quad (3)$$

Different from this calculation, we will calculate in section 2.b the field being emanated in sum, if we take the finite speed of propagation of the fields into account.

(2.b.) An example of four oscillating point charges and their electrostatic field under the assumption of finite speed of propagation, namely of propagation with the speed of light.

For this purpose, we have to describe the positions of our four electrical charges as a function of time explicitly as it was already plotted in fig.1 used as an input for the evaluation of equation (3):

$$s_1(t) = -\frac{d_1 + d_2}{2} + \frac{d_2 - d_1}{2} \cdot \cos\left(2\pi \cdot \frac{t}{\tau}\right) \quad (4)$$

$$s_2(t) = -\frac{d_1 + d_2}{2} - \frac{d_2 - d_1}{2} \cdot \cos\left(2\pi \cdot \frac{t}{\tau}\right) \quad (5)$$

$$s_3(t) = +\frac{d_1 + d_2}{2} + \frac{d_2 - d_1}{2} \cdot \cos\left(2\pi \cdot \frac{t}{\tau}\right) \quad (6)$$

$$s_4(t) = +\frac{d_1 + d_2}{2} - \frac{d_2 - d_1}{2} \cdot \cos\left(2\pi \cdot \frac{t}{\tau}\right) \quad (7)$$

On this basis, the field strength caused by each of the four charges ($\vec{E}_1, \dots, \vec{E}_4$) at the position of the observer r was calculated in the same way as it was done in equation (3). But now, the time at which the field reaches the observer was additionally calculated, taking the moment of emanation of the field into account as well as the duration which the field needs to propagate from its source to the observer with speed of light c . The duration of propagation is simply

$$\Delta t_i = \frac{r - s_i(t)}{c} \quad (\text{with } i = 1 \dots 4) \quad (8)$$

Performing this calculation continuously during time and summing up for each moment all the field strength reaching the observer at this moment (individually calculated for each of the four electrical charges), we receive the total electrical field at the position of the observer as a function of time. In this way, Coulomb's law with additional consideration of the finite speed of propagation of the fields was used.

An exemplary result of this calculation is shown in Fig.3. Now the total field is not zero, which is plausible, because those values of the field strength which compensate each other in equation (3) reach the observer at different moments because of the different time of propagation. On the other hand, those field strength values reaching the observer at the same moment have to pass different distances, and consequently have different absolute values because of Coulomb's law, which will not make it possible any further, that they compensate each other as they did in equation (3)

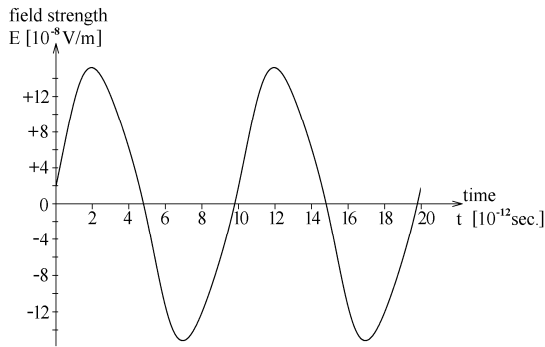


Fig. 3:

Example of the electrostatic field strength generated by four charges oscillating as given in fig.2 and equations (4...7), following Coulomb's law with taking additionally the finite speed of propagation of the fields into account.

The parameters for this example have the values: $d_1 = 0.5\text{m}$, $d_2 = 3.5\text{m}$, $\tau = 10^{-7}\text{ sec}$, $r = 10\text{ m}$, $c = 2.99792458 \cdot 10^8 \frac{\text{m}}{\text{s}}$, $\epsilon_0 = 8.854187817 \cdot 10^{-12} \frac{\text{A}\cdot\text{s}}{\text{V}\cdot\text{m}}$, $q = 1.60217653 \cdot 10^{-19}\text{ C}$.

The calculations have been performed on the basis of SI-units.

Obviously the field strength at the position of the observer is not zero. This is the first and most important result of this article.

But there is another observation: The field strength as a function of time does not exactly follow a sine or cosine. Its deviation from a sine can be seen in equation (9), where the first sine-term gives the dominant part of the field strength, and the next both sine-terms give the continuation of a series (which differs from a Fourier-series because of the individual phase of every sine-term). By the way: The maximum of the speed, with which the electrical charges move in our example is a bit less than $2 \cdot 10^8 \frac{\text{m}}{\text{s}}$. If the charges move slower, the difference between the shape of $E(t)$ and a sine decreases.

The field strength is (calculated with an accuracy of $\Delta E = \pm 3 \cdot 10^{-14} \frac{\text{V}}{\text{m}}$)

$$E(t) = a_0 \cdot \sin\left(1 \cdot \frac{2\pi}{\tau} \cdot (t - a_1)\right) + a_2 \cdot \sin\left(3 \cdot \frac{2\pi}{\tau} \cdot (t - a_3)\right) + a_4 \cdot \sin\left(5 \cdot \frac{2\pi}{\tau} \cdot (t - a_5)\right) + \dots \quad (9)$$

with the coefficients $a_0 = -1.47671114257737 \cdot 10^{-11} \frac{\text{V}}{\text{m}}$, $a_1 = 4.68214978523477 \cdot 10^{-8}\text{ s}$,
 $a_2 = -9.46983556843000 \cdot 10^{-13} \frac{\text{V}}{\text{m}}$, $a_3 = 5.65583264190749 \cdot 10^{-8}\text{ s}$,
 $a_4 = +6.47000984663877 \cdot 10^{-14} \frac{\text{V}}{\text{m}}$, $a_5 = 4.67175185668795 \cdot 10^{-8}\text{ s}$.

Resumé:

There are only two very plausible assumptions which lead to the conclusions of this article:

- the validity of Coulomb's law (following Electrodynamics)
- the fact that fields do not propagate faster than with the speed of light (following the Theory of Relativity)

For these assumptions look very plausible, the consequences are very likely to be as they are described here. Because of this, the author hopes, that this article might encourage experimentalists, to verify the said consequences in experiments. For this purpose, the author will be glad about experimentalists with whom he can plan measurements. It is no problem to adjust the values of the parameters in fig.3 for the requirements of an experiment.

Additional remarks:

- In this article only electrostatic fields are under consideration (even if they alter during time). Magnetic fields are not a subject of this article, but the author plans for future work to take them additionally into account.
- Coulomb's law without consideration of the finite speed of propagation of the field can be understood as a limes of Coulomb's law with consideration of the finite speed of propagation of the field, for $\dot{s}(t)=0$, this is a limes of the speed of the movement of the electric charges coming to zero.

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