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A PRELIMINARY STUDY OF A FUZZY CLUSTERING AND ASSIGNMENT PROBLEM-BASED CELL FORMATION ALGORITHM

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ABSTRACT:
The ideal situation of CM is achieved when each part-cluster together with the machine-cluster for the manufacturing cell is independent from the rest of manufacturing cell, that is exceptional elements are non existent. However, such a situation is rarely achieved. In this paper, a cell formation algorithm that identifies part-clusters and machine-clusters separately is introduced. The algorithm is based on fuzzy clustering and the assignment technique. Computational experience shows that this new algorithm can give superior performance, especially for higher density machine-part incidence matrices.

KEYWORDS:
Cellular Manufacturing, Manufacturing Cell, Cell Formation, Part-cluster, Machine-cluster.

1. INTRODUCTION

GT in particular is a manufacturing-related philosophy that is gaining [1]. This philosophy exploits the similarity among the attributes of given objects [2]. One of the popular applications of GT is called *cellular manufacturing* (CM). In CM, parts that require similar machines are grouped into the same *part-cluster*, while machines needed by a part-cluster are grouped to form a *machine-cluster.* A *manufacturing cell* is part of a manufacturing system where a machine-cluster is placed to process a part-cluster. The most distinctive feature of CM is that it contains dissimilar machines that are located in one area.

The first problem which must be addressed when considering a cellular manufacturing system is that of *cell formation*. Data for cell formation is organized into a *machine-part incidence matrix*. This is a binary matrix with 0 or 1 entries. A 1(one) entry in row *i* and column *j* of the matrix indicates that machine-type *i* needs to operate on part-type *j*, while a 0(zero) indicates it does not. The cell formation problem in CM is the partition or decomposition of the manufacturing system into (manufacturing) cells. Mathematically, the problem involves converting the machine-part incidence matrix into a block diagonal form in which 1(one) entries are concentrated in blocks along the diagonal of the matrix. Each block represents a manufacturing cell [3].

Extensive work has been done by many researchers to provide new techniques for solving this problem. The cell formation problem is complicated by the existence of exceptional parts and/or exceptional machines [4]. An *exceptional part* is a part that requires processing in another machine-cluster. An *exceptional machine* is a machine that processes parts from a different part-cluster. Both exceptional parts and exceptional machines cause *intercellular movement* of parts. Ideally a part-cluster is processed in a single machine cell for its entire operations. In practice, however, it is a very rare case [5]. Thus, the more 1(one) entries are concentrated in the block diagonal the more effective a cell formation technique is. Numerous research papers have appeared in the literature for cell formation. These methods are based on the following approaches: coding and classifications, machine-component analysis, similarity coefficients, mathematical programming and heuristic methods, knowledge-based and pattern recognition methods, neural networks and fuzzy clustering [2].

It is the last approach (ie. fuzzy clustering) that we focus on in this article. Very few papers have appeared in the area of fuzzy clustering techniques applied to cell formation. The application of fuzzy clustering techniques in cellular manufacturing originally proposed by Chu and Hayya [6] and was followed by Ponnambalam and Aravindan [7] with a minor modification in the initialization. Despite their contributions in introducing the application of this technique, there is still a room for improvement, especially in the effort of increasing the 1(one) entries to be concentrated in the block diagonal of the initial given machine-part incidence matrix. The main feature of this paper is to demonstrate a new algorithm to effect this improvement as outlined in Section 3. In Section 2, the basic techniques of the algorithm are discussed. Those techniques are *the fuzzy c-means* (fuzzy clustering) and *the assignment problem.* In Section 4 some examples of computational results are presented. A summary and suggestions for further research are presented in Section 5.

2. A BRIEF REVIEW

2.1 Fuzzy Clustering Problem

Clustering is a process which is common and basic to human understanding. This process, which plays a vital role in grouping of related objects, can be found in such diverse fields as statistics, economics, physics, psychology, biology, pattern recognition, engineering, marketing [8,9]. The primary objective of *clustering* is to partition a given set of objects into so-called homogeneous clusters (groups) [10-12]. Globally, the clustering problem can be divided into two main categories ie. *hard clustering*, in which an object belongs only to one cluster, and *fuzzy clustering*, in which every object belongs to all clusters with different degrees of membership. Definitions, theorems and algorithms involved in the discussion of the fuzzy clustering topic are as follows [13].

2.1.1 Definition and Theorem

Definition 1. Fuzzy c-Partitions

Let $O=\{o_1, o_2, \ldots, o_p\}$ be a set of p objects; $X=\{x_1, x_2, \ldots, x_p\}$ where $x_r=(x_{1r}, x_{2r}, \ldots, x_{mr})^T$ is the *attribute vector* of object $o_r \in O$ (*r* = 1, 2, ..., p); and V_{pc} is the set of real p x c matrices;

Fuzzy c-Partitions space for X is the set

$$M_{fc}=\left\{U \in V_{pc} \middle| u_{ri} \in [0,1] \forall r, i; \sum_{i=1}^{c} u_{ri}=1 \forall r; 0<\sum_{r=1}^{p} u_{ri}<p \forall i\right\} \quad (2.1)$$

Definition 2. Fuzzy c-Means Functional

Fuzzy c-Means Functional is the functional $J_f : M_{fc} \times R^{pc} \rightarrow R^+$ defined by the relationship

$$J_f(U, v)=\sum_{r=1}^{p}\sum_{i=1}^{c}(u_{ri})^f (d_{ri})^2 \quad (2.2)$$

where

1) $U \in M_{fc}$ is a fuzzy c-partitions of X;
2) $v=(v_1, v_2, \ldots, v_c) \in R^{mc}$ with $v_i=(v_{1i}, v_{2i}, \ldots, v_{mi})^T \in R^m$ is the cluster centre of $u_i=(u_{1i}, u_{2i}, \ldots, u_{pi})^T$, $1 \le i \le c$;
3) $d_{ri}=\| x_r - v_i \|$ and $\| \;\; \|$ is any inner product on R^m
4) f is the weighting exponent (also called *the degree of fuzziness*), $f \in (1, \infty)$.

Theorem 1. (Bezdek's Necessary Conditions for Global Minimum)

Let $\| \;\; \|$ be any inner product norm on R^h; $f \in (1,\infty)$; X has at least c (c < p) distinct points, and for each $r \in \{1, 2, \ldots, p\}$, define the sets

$$I_r=\{i \mid 1 \le i \le c; d_{ri}=\|x_r - v_i\|=0\} \quad (2.3)$$

$$I_r^c=\{1, 2, \ldots, c\}-I_r \quad (2.4)$$

then if $(U,v) \in M_{fc} x R^{pc}$ is the global minimum point for J_f then

1) $$I_r=\varnothing \Rightarrow u_{ri}=\frac{1}{\left[\sum_{k=1}^{c}\left(\frac{d_{ri}}{d_{rk}}\right)^{\frac{2}{f-1}}\right]}, \quad (2.5)$$

or

$$I_r=\varnothing \Rightarrow u_{ri}=0 \;\forall i \in I_r^c \text{ and } \sum_{i \in I_r} u_{ri}=1; \quad (2.6)$$

2) $$v_i=\frac{\sum_{r=1}^{p}(u_{ri})^f x_r}{\sum_{r=1}^{p}(u_{ri})^f} \quad \forall i \in \{1, 2, \ldots, c\} \quad (2.7)$$

2.1.2 Chu and Hayya's (1991) Algorithm

Chu and Hayya [6] were the first authors to apply the fuzzy clustering technique to cellformation. They have treated the attribute vector of all part type as an input to their algorithm. Their algorithm will result in two different matrices. The first matrix, notated as U, *the degree of membership matrix*, is used for grouping parts into part-clusters. The second matrix, notated as v, *the cluster centre matrix*, is used for grouping machines into machine-clusters. The detail of their algorithm is as follows.

Let $X = (x_{qr})_{mxp}$ be the given machine-part incidence matrix of *m* types of machine and *p* types of parts; $x_r = (x_{1r}, x_{2r}, \ldots, x_{mr})^T$, namely the *r*-th column of X (*r* = 1,2, ..., p) is the *attribute vector of* the part type *r* . The *p* part types and the *m* machine types are to be grouped into *c* clusters.

Algorithm 1. Chu and Hayya's Algorithm for part-cluster and machine-cluster formation

Step 1 : Fix c, $2 \le c < \min\{m, p\}$; Choose any positive value ξ for stopping criterion;. Choose any inner product norm $\| \ \|$ on R^m and fix $f \in (1, \infty)$; initialize $U^{(0)} \in M_{fc}$ Set $l = 0$.

Step 2 : Calculate the c fuzzy cluster centres $\{v_{(i)}{}^{(l)}\}$, where

$$v_{qi}{}^{(l)} = \frac{\sum_{r=1}^{p} (u_{ri}{}^{(l)})^f x_{qr}}{\sum_{r=1}^{p} (u_{ri}{}^{(l)})^f} \ ; \ i = 1, 2, \ldots, c; \ q = 1, 2, \ldots, m \tag{2.8}$$

Step 3 : $l \leftarrow l + 1$; calculate

$$u_{ri}{}^{(l)} = \begin{cases} \dfrac{1}{\left[\sum_{k=1}^{c} \left(\frac{d_{ri}}{d_{rk}} \right)^{\frac{2}{f-1}} \right]}, & if\ I_r = \varnothing \\ 0, \ \forall k \in I_r^c & if\ I_r \neq \varnothing \\ \dfrac{1}{|I_r|}, \ \forall k \in I_r & if\ I_r \neq \varnothing \end{cases} \tag{2.9}$$

where I_r and $I_r{}^c$ are as defined in (2.4) and (2.5) respectively, $|I_r|$ is the number of element(s) in I_r .

Step 4 : If $\|U^{(l+1)} - U^{(l)}\| < \xi$ then stop else goto Step 2.

The final matrices U and v are used to determine the part-clusters and machine-clusters respectively, under the following rule:

Rule 1. (Part-clusters formation Procedure)

If $u_{ri} = \max_k \{u_{rk}\}$; *r* = 1, 2, ..., *p* (*p* is the number of part-types) and *k* = 1,2, ..., *c* (*c* is the number of part clusters), then part *r* is assigned to part-cluster *i*.

Rule 2. (Machine-clusters formation Procedure)

If $v_{qi} = \max_k \{v_{qk}\}$; *q* = 1, 2, ..., *m* (*m* is the number of machine-types) and *k* = 1,2, ...,*c* (*c* is the number of machine-clusters) then machine *q* is assigned to machine-cluster *i*.

As a result of Chu and Hayya's stopping criterion, two problems may arise. The *first problem*, there is a distinct possibility that their algorithm results in some empty part-clusters and/or empty machine-clusters (ex. see result displayed in Table 3. in [7]). Furthermore, their algorithm may end up with different numbers of part-clusters compared to the resulting number of machine-clusters,which in turn, does not have any practical meaning. The *second problem*, in case of a tie in Rule-1, that is $\max_k \{u_{rk}\}$ is achieved by more than one part-cluster, then in Chu and Hayya's algorithm [6] (or its minor modification by Ponnambalam and Aravindan [7]) the part-type *r* is directly assigned to the first part-cluster achieving the maximum value of u_{rk}. This is too restrictive in the authors' opinion, since it eliminates the opportunity for the other part-cluster that achieved the same maximum value of u_{rk} to contain that particular part type. Improvements to overcome these problems is discussed in Section 3.

2.2 The Assignment Problem

Consider the situation of assigning *c* part-clusters to *c* machine-clusters and define $c_{ik} = \sum_{r \in PC-k} \sum_{q \in MC-i} x_{qr}$ be *the degree of conformance* of part-cluster(PC) *k* and machine-cluster (MC) *i*. That is, the total number of '1'(one) entries in the block diagonal formed by those part and machine-clusters. The objective is to assign the *c* part-clusters to the *c* machine-clusters (one part-cluster per machine-cluster) to maximize the total degree of conformance. This situation is known as *the assignment problem*. (see also [14=15]).

The assignment problem, if applied in cell formation situations, can be expressed in mathematical model as follows. Let

$$\Omega_{ij} = \begin{cases} 0, & \text{if the part} - \text{cluster(PC) } i \text{ is not assigned to the machine} - \text{cluster(MC) } j \\ 1, & \text{if the part} - \text{cluster(PC) } i \text{ is assigned to the machine} - \text{cluster(MC) } j \end{cases} \quad (2.10)$$

The model is thus given by

$$\text{maximize } z = \sum_{i=1}^{c} \sum_{j=1}^{c} c_{ij} \Omega_{ij} \quad (2.11)$$

subject to

$$\sum_{j=1}^{c} \Omega_{ij} = 1, \quad i = 1, 2, \ldots, c \quad (2.12)$$

$$\sum_{i=1}^{c} \Omega_{ij} = 1, \quad j = 1, 2, \ldots, c \quad (2.13)$$

$$\Omega_{ij} = 0 \text{ or } 1. \quad (2.14)$$

Having obtained the part-clusters and the machine-clusters, the application of the assignment problem to the cell formation problem will maximize the total number of elements in the block diagonals of the final permuted machine-part incidence matrix. Thus, at the same time, this will minimize the incidence of exceptional parts and machines.

3. THE SKP-1 (Susanto-Kennedy-Price Version 1) ALGORITHM

The SKP-1 algorithm is aimed at tackling the two problems identified in Chu and Hayya's algorithm [6] and at maximizing the total number of elements in the block diagonal of the final permuted machine-part incidence matrix. As classified by Wemmerlöv [16] (in [17]), the cell formation literature can be divided into four categories, according to the formation logic used: **1**) grouping part-clusters only; **2**) forming part-clusters and then machine-clusters or vice versa; **3**). forming part-clusters and machine-clusters simultaneously, and **4**) grouping machine-clusters only.

The SKP-1 algorithm belongs to the third category in the forementioned classification, and can be outlined as follows. Let X = $(x_{qr})_{mxp}$ be the given machine-part incidence matrix of *m* types of machine and *p* types of parts; $x_r = (x_{1r}, x_{2r}, \ldots, x_{mr})^T$, namely the *r*-th column of X (*r* = 1,2, ..., p) is the *attribute vector of* the part type *r* . Let $y_q = (x_{q1}, x_{q2}, \ldots, x_{qp})$, namely the *q*-th row of X (*q* = 1,2, ..., m) is the *attribute vector of* the machine type *q*. The *p* part types and the *m* machine types are to be grouped into *c* clusters ($2 \le c < \min\{m, p\}$) separately using the fuzzy clustering approach.

3. 1 The First Step of SKP-1 (Machine-clusters Formation)

The *p* attribute vectors of each part type (ie. $x_1, x_2, \ldots, x_p$) serve as the only known variable in the functional to be minimized $J_f(U,v)$ (as defined in (2.2)). In this minimization problem, we define a *successful machine-cluster formation solution* as one that **1**) converges to *c* non empty machine-clusters; **2**) results in unimodal solution for $\max_k\{v_{qk}\}$ (*q* = 1, 2, ..., *m* and *k* = 1, 2, ..., *c*).

Instead of using the criterion in Step-4 of Algorithm-1, we will use a given number of maximum iterations. Thus, up to the maximum number of iterations, it is possible to get more than one *successful machine-cluster formation solution*. The details of the first step in SKP-1 are as follows:

Step 1 : Fix c, $2 \le c < \min\{m,p\}$; choose any inner product norm $\| \ \|$ on R^m and fix $f \in (1,\infty)$;. initialize $U^{(0)} \in M_{fc}$. let *success* = 0; l = 0 and the number of maximum iterations be *max_iterations*.

Step 2 : Calculate the c fuzzy cluster centres $\{v_{(i)}^{(l)}\}$ using (2.8).

Step 3 : If $\forall q \in \{1,2,\ldots,m\}$, the set $A_q = \left\{ \alpha_l^{(q)} \middle| v_{q\alpha_l(k)} = \max_{i=1,2,\ldots,c} \{v_{qi}^{(l)}\} \right\}$ is singleton

then if $\bigcup_{q=1}^{m} A_q = \{1,2,\ldots,c\}$

then

if l < *max_iterations*

then **1**) *success* ← *success* + *1* **2**) at the l-th iteration and at the *success*-th successful machine assignment, assign machine q to machine-cluster $\alpha_l^{(q)}$

else **1**) *success* ← *success* + *1* **2**) at the l-th iteration and at the *success*-th successful machine assignment, assign machine q to machine-cluster $\alpha_l^{(q)}$ **3**) goto Step 6

else if l = *max_iterations* then goto Step 6

else if l = *max_iterations* then goto Step 6

Step 4 : Determine I_r and I_r^c as defined in (2.3) and (2.4), where r = 1, 2, ..., p

Step 5 : $l \leftarrow l + 1$; calculate $\{u_{ri}^{(l)}\}$ as defined in (2.9), goto Step 2.

Step 6 : If $success \ge 1$ then $MC = \bigcup_{s=1}^{success} MC_s$,where $MC_s = \{MC_{is} | i = 1,2,\ldots,c;\ s = 1,2,\ldots, success\}$ and MC_{is} is the machine-cluster i obtained from the s-th successful machine assignment, else "this algorithm fails to converge to a successful machine-cluster formation solution".

3.2 Second Step of SKP-1 (Part-clusters Formation)

Basically the second step of SKP-1 is congruent to that of the first step. While the first step of SKP-1 exploits the part-attribute vectors x_r (r = 1, 2, ..., p) (which are column-vectors) to form the machine-clusters, the second step exploits the machine-attribute vector y_q (q = 1, 2, ..., m) (which are row-vectors) to form the part-clusters. We define a *successful part-cluster formation solution* as one that **1**) converges to c non-empty part-clusters and **2**) results in a unimodal solution for $\max_k \{v_{qk}\}$ (k = 1, 2, ..., c).

Again, instead of using the stopping criterion in Step-4 of Algorithm-1 (Section 2.1.2) we will use a given number of maximum iterations. Thus, up to the maximum number of iterations, it is possible to get more than *one successful part-cluster formation solution*. If, after the maximum number of iterations achieved, this second step converges and results in *SUCCESS successful part-cluster formation solutions* (*SUCCESS* ≥ 1), then let us define:

$PC = \bigcup_{S=1}^{SUCCESS} PC_S$,where $PC_S = \{PC_{kS} | k = 1,2,\ldots,c;\ S = 1,2,\ldots, SUCCESS\}$ and PC_{kS} is the part-cluster k obtained from the S-th successful part-cluster assignment.

3.3 The Third Step of SKP-1 (The Assignment Problem)

To apply this step, it is necessary that both the first and the second step of SKP-1 converge to *success* successful machine-cluster solutions and *SUCCESS* successful part-cluster solutions. Let

$$c_{MC_{is}PC_{kS}} = \sum_{r \in PC_{kS}} \sum_{q \in MC_{is}} x_{qr} \qquad (2.15)$$

be the *degree of conformance* of part cluster k obtained from the S-th successful part-cluster formation and machine cluster i obtained from the s-th successful machine-cluster formation.

The final assignment problem is the solution of the following optimization problem:

$$\text{maximize } z_{sS} = \max \sum_{i=1}^{c} \sum_{k=1}^{c} c_{MC_{is}PC_{kS}} \Omega_{MC_{is}PC_{kS}} \qquad (2.16)$$

subject to

$$\sum_{k=1}^{c} \Omega_{MC_{is}PC_{kS}} = 1, \; i=1,2,\ldots,c; s=1,2,\ldots, success; S=1,2,\ldots, SUCCESS \qquad (2.17)$$

$$\sum_{i=1}^{c} \Omega_{MC_{is}PC_{kS}} = 1, \quad k = 1,2,\ldots,c; \; s=1,2,\ldots, success; S=1,2,\ldots, SUCCESS \qquad (2.18)$$

$$\Omega_{MC_{is}PC_{kS}} = 0 \text{ or } 1 \qquad (2.19)$$

Note: $\Omega_{MC_{is}PC_{kS}} = 1$, if the k-th part-cluster (obtained from the S-th part-cluster formation solution) is assigned to the i-th machine-cluster (obtained from the s-th machine-cluster formation solution), and = 0, otherwise.

4. COMPUTATIONAL RESULTS

In this section the performances of SKP-1 and Chu and Hayya's algorithm are compared, based on the observations applied to 16 matrices with the dimension of 40x50, with density ranges from 20 to 80%. The results are presented in Table 1.

Table-1. Comparison of percentage of non exceptional elements resultedfrom SKP-1 and Chu and Hayya's(1991) algorithm

MATRIX	Density of Matrix *)	SKP -1	CHU & HAYYA [6]	Comment on SKP-1
	20 – 40%			
I	0.207	37.7	35.7	better
II	0.236	38.3	31.6	better
III	0.254	32.3	34.5	worse
IV	0.260	36.0	42.5	worse
V	0.295	25.8	37.4	worse
VI	0.352	29.6	30.7	worse
	40 – 60%			
VII	0.404	31.4	30.4	better
VIII	0.431	28.3	26.0	better
IX	0.445	28.2	29.4	worse
X	0.459	31.8	24.8	better
XI	0.485	35.5	25.6	better
	60 – 80%			
XII	0.611	26.1	25.6	better
XIII	0.659	33.4	29.5	better
XIV	0.669	29.5	23.0	better
XIII	0.704	28.8	24.8	better
XV	0.730	24.4	30.2	worse
XVI	0.805	25.6	20.7	better

*) Density of a matrix as defined in [19]

4.1 Example of Computational Result

The machine-part incidence matrix XVI is displayed in Appendix 1. The results of using this matrix to Chu and Hayya's [6] and the SKP-1 algorithm are outlined in the following Section.

4.1.1 Result from Chu and Hayya's algorithm [6]

This algorithm was applied to matrix XVI with c = 6, f = 2, $\xi = 0.001$ and Euclidean norm as the inner product. The machine-clusters and part-clusters are as follows.

The Machine-clusters :
MC - 1 = {4,7,15,17,35,37}; MC - 2 = {6,19,20,21,23,24,30,31,38,40}; MC - 3 = {8,10,36}; MC - 4 = {3,22};
MC - 5 = {1,5,9,11,12,14,25,26,27,32,33,39} and MC - 6 = {2,13,16,18,28,29,34}.

The Part-clusters :
PC-1 = {1,7,18,19,20,25,33,37,43,49}; PC-2 = {14,32,38,50}; PC-3 = {5,9,21,31,39,40,45}; PC-4 = {4,10, 22, 28,44,46,47}; PC-5 = {2,11,15,17,23,24,26,27,29,34,35,41} and PC-6 = {3,6,8,12,13,16,30,36,42,48}

The resulted Manufacturing Cells are :
Manufacturing Cell-*i* will consist of MC-*i* that processes PC-*i* (*i* = 1,2, …, 6).

There will be 333 of '1'(one) entries on the block diagonal of the final rearrangement of the matrix XVI which contains 1609 (ie.≈0.805x40x50) '1' entries. Thus, Chu and Hayya's [6] algorithm results in 20.7% (ie. 333/1609 x 100%) of non exceptional elements. The final rearrangement by this algorithm is displayed in the Appendix 2.

4.1.2 Result from the SKP-1 algorithm

The SKP-1 algorithm was also applied to matrix XVI, c = 6, f = 2, $\xi = 0.001$, *max_iterations* = 200, and Euclidean norm as the inner product. Four successful machine-cluster formation solutions and five successful part-cluster formation solutions resulted from the treatment are:

The Four Successful Machine-cluster Formation Solutions :

The first successful machine-cluster formation solution:
MC - 1 = {1,2,3,4,7,15,17,35,37}; MC - 2 = {6,19,23,25,30,31,38,40}; MC - 3 = {8,10,18,21,24,36}; MC - 4 = {20,22,27}; MC - 5 = {5,9,11,14,26,32,33,39} and MC - 6 = {12,13,16,28,29,34}.

The second successful machine-cluster formation solution:
MC - 1 = {1,3,4,7,15,16,17,35,36,37,40}; MC - 2 = {6,19,23,31,38}; MC - 3 = {8,10,21,24,30}; ;
MC - 4 = {20,22}; MC - 5 = {5,9,11,14,25,26,27,32,33,39} and MC - 6 = {2,12,13,18,28,29,34}

The third successful machine-cluster formation solution:
MC- 1 = {15,16,17,20,35,36,37,38,40}; MC - 2 = {19,31}; MC - 3 = {7,8,10,12,24,30};MC - 4 = {3};
MC - 5 = {1,5,9,14,21,22,25,26,27,32,33,39} and MC - 6 = {2,4,6,11,13,18,23,28,29,34}

The fourth successful machine-cluster formation solution:
MC-1 = {15,16,18,20,31,35,36,37,38,40}; MC-2 = {19}; MC -3 = {7,8,10,12,17,24,30}; MC-4 = {3}
MC - 5 = {1,5,9,14,21,22,25,26,27,32,33,39} and MC - 6 = {2,4,6,11,13,23,28,29,34}

The Five Successful Part-cluster Formation Solutions :

The first successful part-cluster formation solution:
PC- 1 = {6,7,13,22,25,32}; PC- 2 = {34,35,46,47,49}; PC- 3 = {10,14,15,17,26,27,30,31,43,44,45}; PC- 4 = {1,8,12,16,18,19,20,21,23,28,33,36,37,38,48,50}; PC- 5 = {11} and PC- 6 = {2,3,4,5,9,24,29, 39,40,41,42}

The second successful part-cluster formation solution:
PC- 1 = {6,13,18,25,32}; PC- 2 = {34,35,47,49}; PC- 3 = {10,14,15,17,26,27,31,43,45}; PC- 4 = {1,7,8,12,16, 19, 20,21,22,23,28,33,36,37,38,42,48,50}; PC- 5 = {11,30,46} and PC- 6 = {2,3,4,5,9,24,29,39,40,41,44}

The third successful part-cluster formation solution:
PC- 1 = {4,6,7,13,18,22,25,32,38}; PC- 2 = {34,49}; PC- 3 = {10,14,15,17,26,27,29,31,43,45,47}; PC- 4 = {1,8,12,16,19,20,21,23,28,33,35,36,37,42,48,50}; PC- 5 = {11,30,41,46}; PC- 6 = {2,3,5,9,24,39,40,44}

The fourth successful part-cluster formation solution:
PC- 1 = {4,7,13,18,22,32,38}; PC- 2 = {3,34,49}; PC- 3 = {6,10,14,15,17,25,26,27,29,31,43,47}; PC- 4 = {1,8, 12,16,19,20,21,23,28,33,35,36,37,39,42,48,50}; PC- 5 = {2,11,24,30,41,45,46} and PC- 6 = {5,9,40,44}

The fifth successful part-cluster formation solution:
PC- 1 = {4,7,18,22,32,38}; PC- 2 = {49}; PC- 3 = {6,9,10,14,15,17,25,26,27,29,31,34,43,47}; PC- 4 = {1,8,12, 16,19,20,21,23,28,33,35,36,37,39,42,48,50}; PC- 5 = {2,3,11,13,24,30,41,45,46} and PC- 6 = {5,40,44}

The desired machine-clusters and part-clusters are obtained by pairing each successful machine-cluster and part-cluster formation solution, and applying the Assignment Algorithm to each pair. In the case of machine-part incidence matrix XVI, the desired machine-clusters are those obtained from the *fourth* successful machine-cluster formation solution and the desired part-clusters are those obtained from the *fourth* successful part-cluster formation problem.

The resulted Manufacturing Cells are :
M-Cell 1 = (MC-1, PC-4); M-Cell 2 = (MC-2, PC-6); M-Cell 3 = (MC-3, PC-1); M-Cell 4 = (MC-4, PC-2); M-Cell 5 = (MC-5, PC-3) and M-Cell 6 = (MC-6, PC-5).

There will be 412 of 1(one) entries on the block diagonal of the final rearrangement of the matrix XVI out of 1609 entries. Thus, this algorithm results in 25.6% of non exceptional elements. The final rearrangement by this algorithm is displayed in the Appendix 3..

5. SUMMARY AND SUGGESTIONS

In general, computational experience summarized in Table 1 shows that for higher density machine-part incidence matrix, the proposed new algorithm, ie. Susanto-Kennedy-Price Version 1 or SKP-1, has more chance in giving cell formation solutions with fewer numbers of exceptional elements.

However, further research is required to identify other attributes of the input machine-part incidence matrix, such as *the total bond energy* matrix (as defined in [19]), in causing exceptional elements in the final solution.

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APPENDICES

Appendix 1. The Machine-Part Incidence Matrix XVI

Machine Type Number	Part-type Number
	0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 4 4 5
	1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0
01	1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 0 1 1 1 1 0 1 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1
02	1 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 0 1 1 1 1 1 1
03	0 0 0 1 1 1 1 0 1 1 0 1 1 1 1 0 1 1 1 1 0 1 0 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1
04	1 1 0 0 1 0 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 0 1 1 1 0 1 1 0 0 0 1 1 0 1 0 1 1 1 1 1 1 1 0 0 1 0 1 1 1
05	0 1 0 1 1 1 0 1 0 1 1 0 1 1 1 0 1 1 0 0 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 0 1 0 1 1 1 1 1 1 1 1 1 1
06	1 1 1 1 1 0 0 1 1 1 1 1 1 1 0 1 0 0 1 1 1 1 1 1 0 1 0 0 1 1 1 1 1 0 1 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1
07	1 1 1 1 0 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 0 1 0 1 1 0 1 1 0 1 1 1 0 1 0 1 0
08	1 1 1 0 1 0 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 0 1 1 0 1 1 1 1 0 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 0 0 1 0
09	1 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 0 0 1 1 0 0 0 1 1
10	1 0 1 1 1 1 1 1 1 1 0 1 0 1 1 1 0 1 0 1 1 1 1 1 1 1 1 0 0 1 1 1 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0
11	1 1 1 1 1 0 1 0 1 0 1 1 1 1 0 1 1 0 1 1 1 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1
12	0 1 1 1 1 1 0 1 1 0 0 1 1 0 1 1 1 1 1 1 1 0 1 1 0 1 1 0 1 1 1 0 0 1 1 1 0 1 1 1 1 1 1 0 0 0 1 1 1 0
13	1 1 1 1 1 1 1 1 0 0 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 0 1 0 1
14	0 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0 0 1 0 1 1 0 1 1 1 1 0
15	1 0 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 0 1 1 0 1 0 1 1 1 1 0 1 1 1 1
16	1 0 1 0 1 1 1 1 1 0 0 1 1 1 0 1 1 1 1 1 1 1 1 0 0 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 0 1 0 0 1 0 1 1 1 1
17	1 0 1 1 1 0 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 0 0 1 1 1 1
18	0 1 1 1 1 1 0 1 1 1 0 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 0 1
19	1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1
20	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1 0 1 0 1 0 1 0 1 0 1 1 1 1 1 1
21	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
22	1 1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 1 0 1
23	1 1 1 0 1 0 0 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 0 0 1 0 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
24	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 0 1 1 1 1 0 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 1 0 1 1
25	0 1 0 0 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 0 1 1 1 0 1 1 1 1 1 1 0 1 1
26	1 1 1 0 1 1 0 1 0 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 0 1 0 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1
27	0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 0 1 1 0 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1
28	1 1 1 1 1 1 1 1 0 0 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 1
29	1 1 1 1 0 1 1 1 0 1 1 1 1 1 0 1 1 0 1 1 0 0 1 1 0 1 1 1 1 1 0 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1
30	1 1 1 1 1 0 0 1 1 1 1 1 0 1 1 1 0 0 1 1 1 0 1 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
31	0 1 1 1 0 1 1 1 1 1 0 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1
32	1 1 1 0 1 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 0 1 1 1 0 1 1 1 0 1 1 1 1 1 0
33	1 1 1 0 0 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1
34	1 0 1 1 0 1 1 1 1 1 1 1 1 0 0 1 0 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1
35	1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 0 1 1 1 1 0 1 0 1 1 1 0
36	1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 0 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1
37	1 1 0 1 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 1
38	1 0 1 1 0 1 1 1 0 1 1 1 0 1 1 0 0 1 1 1 0 0 1 1 0 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1
39	1 0 1 1 0 1 1 1 1 1 1 0 1 1 1 0 1 0 0 0 1 0 1 0 0 1 1 1 1 1 1 1 0 1 1 1 0 1 0 1 1 1 1 1 1 1 1 1 1 0
40	1 1 1 1 0 0 1 1 0 0 1 1 0 1 1 1 1 1 1 1 1 1 1 0 0 1 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1

Appendix 2. The Final Rearrangement of Machine-Part Incidence Matrix XVI by Chu and Hayya's Algorithm

Machine Type Number	Part-type Number					
	0 0 1 1 2 2 3 3 4 4	1 3 3 5	0 0 2 3 3 4 4	0 1 2 2 4 4 4	0 1 1 1 2 2 2 2 2 3 3 4	0 0 0 1 1 1 3 3 4 4
	1 7 8 9 0 5 3 7 3 9	4 2 8 0	5 9 1 1 9 0 5	4 0 2 8 4 6 7	2 1 5 7 3 4 6 7 9 4 5 1	3 6 8 2 3 6 0 6 2 8
4	1 1 1 1 1 1 1 1 1 1	1 1 1 1	1 1 1 0 1 1 0	0 0 0 1 0 1 0	1 1 0 1 1 1 0 1 0 0 1 1	0 0 1 1 1 1 0 0 1 1
7	1 1 1 1 1 1 1 1 1 1	1 1 1 0	0 1 1 1 0 1 1	1 1 1 1 1 0 1	1 1 1 1 1 1 1 0 0 0 1 1	1 1 0 1 0 1 1 0 0 0
15	1 1 1 1 1 0 0 1 1 1	1 1 1 1	1 1 1 1 0 1 1	1 1 1 1 1 0 1	0 1 0 1 1 1 1 1 1 1 1 0	1 1 0 1 1 1 1 0 1 1
17	1 1 1 1 1 1 1 1 1 1	1 1 1 1	1 1 1 1 1 1 0	1 1 1 1 1 0 1	0 1 1 1 1 1 1 1 1 1 1 0	1 0 1 1 0 0 1 1 0 1
35	1 1 1 1 1 1 0 1 1 1	1 1 1 0	1 1 1 1 0 1 1	1 1 1 1 0 0 1	0 1 1 1 0 1 1 1 1 1 1 1	1 1 1 1 1 1 0 1 1 1
37	1 1 1 1 1 1 1 1 1 1	1 1 1 1	0 1 1 1 0 1 1	1 1 1 1 1 1 1	1 0 1 1 1 0 1 1 1 1 1 0	0 1 1 1 1 1 0 1 1 1
6	1 0 0 1 1 0 1 0 0 1	1 1 1 1	1 1 1 1 1 1 1	1 1 1 0 1 1 1	1 1 0 0 1 1 1 0 1 0 1 1	1 0 1 1 1 1 1 0 1 1
19	1 1 1 1 1 0 1 1 1 0	1 1 1 1	1 1 1 1 1 1 1	1 1 1 1 1 1 0	0 1 1 0 1 1 1 1 1 0 1 1	0 0 1 1 1 1 1 1 1 1
20	1 1 0 1 1 0 1 0 0 1	1 1 1 1	1 1 1 1 0 1 0	1 1 1 1 1 1 1	1 1 1 0 1 1 0 1 1 1 1 0	1 1 1 1 1 1 1 1 1 1
21	1 1 1 0 1 1 1 1 1 1	1 1 1 1	1 1 1 1 1 1 1	1 1 1 0 1 1 1	1 1 1 1 0 1 1 1 1 1 1 1	1 1 1 1 1 1 1 0 1 1
23	1 0 1 1 1 1 1 1 1 1	1 1 1 1	1 1 1 0 1 1 1	0 1 1 1 1 1 1	1 1 0 0 1 1 0 0 0 1 0 1	1 0 1 1 1 1 1 1 1 1
24	1 1 1 1 1 0 1 0 1 1	1 1 1 1	1 1 1 1 1 1 1	1 1 0 1 1 1 1	1 1 1 1 1 1 1 1 1 1 0 1	1 1 1 1 1 0 0 0 1 0
30	1 0 0 1 1 1 0 1 1 1	1 1 1 1	1 1 1 1 1 1 1	1 1 0 1 1 1 1	1 1 1 0 1 1 1 1 1 1 1 1	1 0 1 1 0 1 0 1 1 1
31	0 1 1 1 1 1 1 1 0 1	1 1 1 1	0 1 1 1 1 1 1	1 1 1 1 1 1 1	1 0 0 0 1 1 1 0 1 1 1 1	1 1 1 1 1 0 0 1 1 1
38	1 1 1 1 1 0 1 1 1 1	1 1 1 1	0 0 0 1 1 1 0	1 1 0 0 1 1 1	0 1 1 0 1 1 1 1 1 1 1 1	1 1 1 1 0 0 1 0 1 1
40	1 1 1 1 1 0 0 1 1 1	1 1 1 1	0 0 1 1 1 1 0	1 0 1 1 1 1 1	1 1 1 1 1 0 1 0 1 1 1 1	1 0 1 1 0 1 1 1 1 1
8	1 1 1 0 1 0 1 0 1 1	1 1 1 0	1 1 1 1 1 1 1	0 1 0 1 1 0 0	1 0 1 1 1 1 1 1 1 1 1 1	1 0 1 1 1 1 0 1 1 0
10	1 1 1 0 1 1 0 1 1 1	1 1 1 0	1 1 1 1 1 1 1	1 1 1 0 1 1 1	0 0 1 0 1 1 1 1 0 0 0 0	1 1 1 1 0 1 1 1 1 1
36	1 1 1 1 1 1 1 1 1 1	1 1 1 1	1 1 1 1 1 1 1	1 1 0 0 1 1 1	1 0 1 1 1 1 0 1 1 1 1 1	1 1 1 1 1 1 1 1 1 0
3	0 1 1 1 1 1 1 1 1 1	1 1 1 1	1 1 0 1 1 1 1	1 1 1 1 1 1 1	0 0 1 1 0 0 1 1 1 1 0 1	0 1 0 1 1 0 1 1 1 0
22	1 0 1 0 1 1 1 1 1 0	1 1 1 1	1 1 1 1 0 1 1	0 1 1 1 1 1 1	1 1 1 1 0 1 1 1 1 1 1 1	0 1 1 1 1 1 1 1 0 1
1	1 1 0 1 1 1 0 1 1 1	1 1 1 1	1 1 1 1 1 0 1	1 1 1 0 1 1 1	1 1 1 1 0 1 1 1 1 1 0 1	1 1 1 0 1 1 1 1 1 1
5	0 0 1 0 0 0 1 1 1 1	1 1 0 1	1 0 0 1 1 0 1	1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1	0 1 1 0 1 0 1 0 1 1
9	1 0 1 0 0 1 0 1 0 1	1 0 1 1	1 1 0 1 1 1 1	1 1 0 1 1 0 0	1 1 1 1 1 1 1 1 1 0 1 1	1 0 1 0 0 1 1 1 0 0
11	1 1 0 1 1 1 1 1 1 1	1 1 0 1	1 1 1 1 1 1 1	1 0 0 0 1 1 1	1 1 0 1 1 1 1 1 1 1 1 1	1 0 0 1 1 1 1 1 1 1
12	0 0 1 1 1 0 0 0 1 1	0 0 1 0	1 1 1 1 1 1 0	1 0 0 0 0 0 1	1 0 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1
14	0 1 1 1 1 1 1 0 1 1	1 1 1 0	1 1 1 1 0 0 0	1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1	1 1 0 1 1 0 1 0 0 1
25	0 1 1 1 1 1 0 0 1 1	1 1 1 1	1 1 0 1 1 1 1	0 0 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 0	0 1 1 0 1 1 1 1 1 0
26	1 0 0 1 0 1 1 1 0 1	1 0 1 1	1 0 1 1 1 1 1	0 1 1 1 1 1 1	1 1 1 1 1 1 1 0 0 1 1 1	1 1 1 1 1 1 1 1 1 1
27	0 1 1 0 1 0 0 1 1 0	1 0 0 1	1 1 1 1 1 1 1	1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 0 1 1 1	1 0 1 1 1 1 1 1 1 1
32	1 1 0 0 0 1 0 0 1 1	1 0 1 0	1 1 1 1 1 0 1	0 1 1 1 0 1 1	1 1 1 1 1 1 1 1 1 1 1 1	1 0 1 1 1 1 1 1 1 1
33	1 0 1 1 1 1 0 0 1 1	1 1 1 1	0 0 1 1 1 1 1	0 1 1 0 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1	1 1 0 1 1 1 1 0 1 1
39	1 1 0 0 0 0 0 0 1 1	1 1 1 0	0 1 1 1 0 1 1	1 1 0 1 1 1 1	0 1 1 1 1 0 1 1 1 1 1 1	1 1 1 0 1 0 1 1 1 1
2	1 1 1 1 1 1 1 1 1 1	1 1 1 1	0 0 1 1 0 0 1	1 1 1 0 0 1 1	1 1 1 1 0 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1
13	1 1 1 1 1 1 0 1 1 0	1 1 1 1	1 0 1 1 1 1 1	1 0 1 0 0 1 0	1 1 0 1 0 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1
16	1 1 1 1 1 0 1 1 0 1	1 1 1 1	1 1 1 1 1 1 1	0 0 1 1 0 0 1	0 0 0 1 1 0 1 1 0 0 1 0	1 1 1 1 1 1 1 1 1 1
18	0 0 1 1 1 1 1 1 1 0	1 1 1 1	1 1 1 1 1 1 1	1 1 1 1 1 0 1	1 0 1 0 0 1 1 1 1 1 1 1	1 1 1 1 1 1 1 0 1 1
28	1 1 1 1 1 1 1 1 1 1	1 1 1 1	1 0 1 0 1 0 1	1 0 1 1 1 1 0	1 1 0 0 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1
29	1 1 0 1 1 1 1 1 1 0	1 0 1 1	0 0 0 0 1 0 1	1 1 0 1 1 1 1	1 1 0 1 1 1 0 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1
34	1 1 1 1 1 0 1 1 1 1	0 1 1 1	0 1 1 1 1 0 1	1 1 1 1 1 1 1	0 1 0 0 1 1 1 0 1 1 1 1	1 1 1 1 1 1 1 1 1 1

Appendix 3. The Final Rearrangement of Machine-Part Incidence Matrix XVI by Susanto-Kennedy-Price Version 1 (SKP-1) Algorithm

Part-type Number

Machine Type Number						
	0 0 1 1 1 2 2 2 2 3 3 3 3 3 4 4 5	0 4 4	0 0 1 2 3 3	4	0 0 1 1 1 1 2 2 2 2 3 3 4 4	0 0 1 1 2 3 4 4 4
	1 8 2 6 9 0 1 3 8 3 5 6 7 9 2 8 0	5 0 4	4 7 8 2 2 8	9	6 9 0 4 5 7 5 6 7 9 1 4 3 7	2 3 1 3 4 0 1 5 6
15	1 0 1 1 1 1 1 1 1 0 1 0 1 0 1 1 1	1 1 1	1 1 1 1 1 1	1	1 1 1 1 0 1 0 1 1 1 1 1 1 1	0 1 1 1 1 1 0 1 0
16	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 0	0 1 1 1 1 1	1	1 1 0 1 0 1 0 1 1 0 1 0 0 1	0 1 0 1 0 1 0 1 0
18	0 1 1 1 1 1 1 0 1 1 1 0 1 1 1 1 1	1 1 1	1 0 1 1 1 1	0	1 1 1 1 1 0 1 1 1 1 1 1 1 1	1 1 0 1 1 1 1 1 0
20	1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1	1 1 1	1 1 0 1 1 1	1	1 1 1 1 1 0 0 0 1 1 1 1 0 1	1 1 1 1 1 1 0 0 1
31	0 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1	0 1 1	1 1 1 1 1 1	1	1 1 1 1 0 0 1 1 0 1 1 1 0 1	1 1 0 1 1 0 1 1 1
35	1 1 1 1 1 1 1 0 1 0 1 1 1 0 1 1 0	1 1 0	1 1 1 1 1 1	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1	0 1 1 1 1 0 1 1 0
36	1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1	1 1 1	1 1 1 0 1 1	1	1 1 1 1 1 1 1 0 1 1 1 1 1 1	1 1 0 1 1 1 1 1 1
37	1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1	0 1 1	1 1 1 1 1 1	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 0 0 1 0 0 0 1 1
38	1 1 1 0 1 1 0 1 0 1 1 0 1 1 1 1 1	0 1 1	1 1 1 0 1 1	1	1 0 1 1 1 0 0 1 1 1 1 1 1 1	0 1 1 0 1 1 1 0 1
40	1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1	0 1 1	1 1 1 1 1 1	1	0 0 0 1 1 1 0 1 0 1 1 1 1 1	1 1 1 0 0 1 1 0 1
19	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1	1 1 1 1 1 1	0	0 1 1 1 1 0 0 1 1 1 1 0 1 0	0 0 1 1 1 1 1 1 1
7	1 0 1 1 1 1 1 1 1 1 1 0 1 0 0 0 0	0 1 1	1 1 1 1 1 1	1	1 1 1 1 1 1 1 1 0 0 1 0 1 1	1 1 1 0 1 1 1 1 0
8	1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 0 0	1 1 1	0 1 1 0 1 1	1	0 1 1 1 1 1 0 1 1 1 1 1 1 0	1 1 0 1 1 0 1 1 0
10	1 1 1 1 0 1 1 1 0 0 0 1 1 1 1 1 0	1 1 1	1 1 1 1 1 1	1	1 1 1 1 1 0 1 1 1 0 1 0 1 1	0 1 0 0 1 1 0 1 1
12	0 1 1 1 1 1 1 1 0 0 1 1 0 1 1 1 0	1 1 0	1 0 1 0 0 1	1	1 1 0 0 1 1 0 1 1 1 1 1 1 1	1 1 0 1 1 1 1 0 0
17	1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1	1 1 1	1 1 1 1 1 1	1	0 1 1 1 1 1 1 1 1 1 1 1 1 1	0 1 1 0 1 1 0 0 0
24	1 1 1 0 1 1 1 1 1 1 0 0 0 1 1 0 1	1 1 1	1 1 1 0 1 1	1	1 1 1 1 1 1 0 1 1 1 1 1 1 1	1 1 1 1 1 0 1 1 1
30	1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1	1 1 1	1 0 0 0 1 1	1	0 1 1 1 1 0 1 1 1 1 1 1 1 1	1 1 1 0 1 0 1 1 1
3	0 0 1 0 1 1 0 0 1 1 0 1 1 1 1 0 1	1 1 1	1 1 1 1 1 1	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1	0 0 0 1 0 1 1 1 1
1	1 1 0 1 1 1 1 0 0 0 0 1 1 1 1 1 1	1 0 1	1 1 0 1 1 1	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1
5	0 1 0 0 0 0 0 1 1 1 1 0 1 1 1 1 1	1 0 1	1 0 1 1 1 0	1	1 0 1 1 1 1 0 1 1 1 1 1 1 1	1 0 1 1 1 1 1 1 1
9	1 1 0 1 0 0 0 1 1 0 1 1 1 1 0 0 1	1 1 1	1 0 1 0 0 1	1	0 1 1 1 1 1 1 1 1 1 1 0 0 0	1 1 1 0 1 1 1 1 0
14	0 0 1 0 1 1 1 1 1 1 1 0 0 0 0 1 0	1 0 1	1 1 1 1 1 1	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 0 1
21	1 1 1 1 0 1 1 0 0 1 1 0 1 1 1 1 1	1 1 1	1 1 1 1 1 1	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1
22	1 1 1 1 0 1 1 0 1 1 1 1 1 0 0 1 1	1 1 1	0 0 1 1 1 1	0	1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 0 1 1 1 1 1 1 1
25	0 1 0 1 1 1 0 1 1 0 1 1 0 1 1 0 1	1 1 1	0 1 1 1 1 1	1	1 1 0 1 1 1 1 1 1 1 1 1 1 1	1 0 1 1 1 1 0 1 1
26	1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1	1 1 1	0 0 0 1 0 1	1	1 0 1 1 1 1 1 1 0 0 1 1 0 1	1 1 1 1 1 1 1 1 1
27	0 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1	1 1 1	1 1 1 1 0 0	0	0 1 1 1 1 1 0 1 1 0 1 1 1 1	1 1 1 1 1 1 1 1 1
32	1 1 1 1 0 0 1 1 1 0 1 1 0 1 1 1 0	1 0 0	0 1 0 1 0 1	1	0 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1
33	1 0 1 1 1 1 1 1 0 0 1 0 0 1 1 1 1	0 1 1	0 0 1 1 1 1	1	1 0 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1
39	1 1 0 0 0 0 1 1 1 0 1 1 0 0 1 1 0	0 1 1	1 1 0 0 1 1	1	1 1 1 1 1 1 0 1 1 1 1 1 1 1	0 1 1 1 0 1 1 1 1
2	1 1 1 1 1 1 1 0 0 1 1 1 1 0 1 1 1	0 0 0	1 1 1 1 1 1	1	1 0 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1
4	1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1	1 1 0	0 1 1 0 1 1	1	0 1 0 1 0 1 1 0 1 0 0 0 1 0	1 0 1 1 1 0 1 0 1
6	1 1 1 1 1 1 1 1 0 1 1 0 0 1 1 1 1	1 1 1	1 0 0 1 1 1	1	0 1 1 1 0 0 0 1 0 1 1 0 0 1	1 1 1 1 1 1 1 1 1
11	1 0 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1	1 1 1	1 1 0 0 1 0	1	0 1 0 1 0 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1
13	1 1 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1	1 1 0	1 1 1 1 1 1	0	1 0 0 1 0 1 1 1 1 1 1 1 1 0	1 1 1 1 1 1 1 1 1
23	1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1	1 1 1	0 0 1 1 1 1	1	0 1 1 1 0 0 1 0 0 0 0 1 1 1	1 1 1 1 1 1 1 1 1
28	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 0 1	1 1 1 1 1 1	1	1 0 0 1 0 0 1 1 1 1 0 1 1 0	1 1 1 1 1 1 1 1 1
29	1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1	0 0 1	1 1 0 0 0 1	0	1 0 1 1 0 1 1 0 1 1 0 1 1 1	1 1 1 1 1 1 1 1 1
34	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	0 0 1	1 1 1 1 1 1	1	1 1 1 0 0 0 0 1 0 1 1 1 1 1	0 1 1 1 1 1 1 1 1